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QUESTION PAPER CODE: X10658

B.E. / B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2020
Third Semester
Electronics and Communication Engineering
MA8352 –LINEAR ALGEBRA AND PARTIAL DIFFERENTIAL EQUATIONS
(Common to Biomedical Engineering, Computer and Communication Engineering, Electronics and Telecommunication Engineering and Medical Electronics)
(Regulations 2017)
Answer ALL Questions

Time: 3 Hours **Maximum Marks:100**

PART A ($10 \times 2 = 20$ Marks)

1. Determine whether the subset $S = \{(x, y, 0)|x \text{ and } y \text{ are real numbers}\}$ of the vector space $V = R^3$ is a subspace or not.

2. For which values of k will the vector $v = (1, -2, k)$ in R^3 be a linear combination of the vectors $u = (3, 0, -2)$ and $w = (2, -1, -5)$.

3. Find the matrix representation of a linear transformation $T : P_3(R) \to P_2(R)$ defined by $T(f(x)) = f'(x)$ with respect to the standard ordered bases for $P_3(R)$ and $P_2(R)$.

4. Is there a linear transformation $T : R^3 \to R^2$ such that $T(1, 0, 3) = (1, 1)$ and $T(-2, 0, -6) = (2, 1)$? Justify.

5. Prove that $||\alpha + \beta|| \leq ||\alpha|| + ||\beta||$ for any two vectors $\alpha,\ \beta$ belong to the standard inner product space.

6. Find the orthogonal complement of $S = (0, 0, 1)$ in an inner product space R^3.

7. Obtain the partial differential equation by eliminating the arbitrary function from $z = f(x^2 + y^2)$.

8. Find the complete solution of $p^2 + q^2 = 4$.

9. State Dirichlet's conditions for a function $f(x)$ to be expressed as a Fourier series.

10. Solve $x^2 \dfrac{\partial z}{\partial x} + y^3 \dfrac{\partial z}{\partial y} = 0$ by method of separation of variables.

PART-B (5×16=80 Marks)

11. (a) (i) Let $V = R^3$ and $S_1 = \{(1, 0, 0), (2, 2, 0), (5, 7, 2)\}$. Show that S_1 is a minimal generating set. (8)

(ii) Verify whether the set $S = \left\{ \begin{pmatrix} 1 & -3 & 2 \\ -4 & 0 & 5 \end{pmatrix}, \begin{pmatrix} -3 & 7 & 4 \\ 6 & -2 & -7 \end{pmatrix}, \begin{pmatrix} -2 & 3 & 11 \\ -1 & -3 & 2 \end{pmatrix} \right\}$ in $M_{2\times3}(R)$ is linearly dependent or not. (6) (8)

(OR)

(b) (i) Let $V = R^3$ and $S_1 = \{(1,0,0),(2,2,0),(5,7,2)\}$. Show that S_1 is a minimal generating set. (8)

(ii) Let $V = R^3$, $W_1 = \{(x,x,x)/x \in R\}$ and $W_2 = \{(0,y,z)/y,z \in R\}$ are two subspaces of V, then prove that $V = W_1 \oplus W_2$. (8)

12. (a) Let $T : R^2 \to R^3$ and $U : R^2 \to R^3$ be the linear transformations defined by $T(a_1,a_2) = (a_1 + 3a_2, 0, 2a_1 - 4a_2)$ and $U(a_1,a_2) = (a_1 - a_2, 2a_1, 3a_1 + 2a_2)$ respectively. Then prove that $[T+U]_B^\gamma = [T]_B^\gamma + [U]_B^\gamma$. (16)

(OR)

(b) Let T be a linear operator on $P_2(R)$ defined by $T[f(x)] = f(1) + f'(0)x[f'(0) + f''(0)]x^2$. Test for diagonalizability. (16)

13. (a) Let $V = P(R)$ with the inner product $< f(x), g(x) >= \int_{-1}^{1} f(t)g(t)dt$. Consider the sub space $P_2(R)$ with standard ordered basis B. Use the Gram-Schmidt process to replace B by an orthogonal basis $\{v_1, v_2, v_3\}$ for $P_2(R)$ and use that orthogonal basis to obtain an orthonormal basis for $P_2(R)$. (16)

(OR)

(b) (i) Let $V = C^3$ where C is the set of complex numbers. Define $< x,y >= a_1\overline{b_1} + a_2\overline{b_2} + a_3\overline{b_3}$ where $x = (a_1,a_2,a_3)$ and $y = (b_1,b_2,b_3)$. Verify whether V is an inner product space or not. (8)

(ii) Let V be a finite dimensional inner product space, and let T be a linear operator on V. Then show that there exists a unique linear function $T^* : V \to V$ such that $< T(x), y >=< x, T^*(y) >$ for all $x, y \in V$. (8)

14. (a) (i) Solve:$(D^2 + DD' - 6D'^2)z = y \cos x$. (8)

(ii) Find the singular integral of $z = px + qy + \sqrt{1 + p^2 + q^2}$. (8)

(OR)

(b) (i) Find the integral surface of the equation $2y(z-3)p + (2x - z)q = y(2x - 3)$ that passes through the circle $x^2 + y^2 = 2x$ and $z = 0$. (8)

(ii) Find the complete solution of $x^2p^2 + y^2q^2 = z^2$. (8)

15. (a) A rectangular plate $0 \le x \le 20,\ 0 \le y \le 10$ has the edges $x = 0,\ x = 20,\ y = 0$ maintained at zero temperature and the edge $y = 10$ has the temperature $u = 20x - x^2$. Find the steady state temperature at any point (x,y) on the plate. (16)

(OR)

(b) (i) Obtain the Fourier series for the function $f(x) = |x|,\ -\pi < x < \pi$. (8)

(ii) Express $f(x) = x(\pi - x),\ 0 < x < \pi$, as a Fourier series of periodicity 2π containing cosine terms only. (8)

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