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**Question Paper Code : 40786**

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2021.

Third Semester

Civil Engineering

MA 8353 – TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

(Common to : Aeronautical Engineering/Aerospace Engineering/  
Agriculture Engineering/Automobile Engineering/Electrical and Electronics  
Engineering/Electronics and Instrumentation Engineering/  
Industrial Engineering/Industrial Engineering and Management/  
Instrumentation and Control Engineering/Manufacturing Engineering/  
Marine Engineering/Material Science and Engineering/Mechanical  
Engineering/Mechanical Engineering (Sandwich)/Mechanical and Automation  
Engineering/Mechatronics Engineering/Production Engineering/Robotics and  
Automation/Bio Technology/Biotechnology and Biochemical Engineering/Chemical  
and Electrochemical Engineering/Food Technology/Pharmaceutical Technology)

(Regulations 2017)

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Form the partial differential equation by eliminating the arbitrary constants 'a' and 'b' from the relation  $z = a(x + y) + b$ .
2. Find the complete solution of  $pq = xy$ .
3. Fourier series of a function  $f(x) = \begin{cases} \pi + x, & -\pi < x < 0 \\ \pi - x, & 0 < x < \pi \end{cases}$  is given by  $\frac{\pi}{2} + \frac{4}{\pi} \left( \sum_{n=1, 3, 5, \dots} \frac{1}{n^2} \cos nx \right)$ . What is the function represented by the same Fourier series the interval  $(\pi, 3\pi)$ ?
4. Find the Fourier series of a function  $f(x)$  up to the first harmonic from the following data  $n = 12$ ,  $\sum f(x) = 50.090$ ,  $\sum f(x) \cos x = 14.699$  and  $\sum f(x) \sin x = 18.962$ .

5. Classify the partial differential equation

$$x^2 \frac{\partial^2 u}{\partial x^2} - 2xy \frac{\partial^2 u}{\partial x \partial y} + (1 + y^2) \frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial x} + 3 \frac{\partial u}{\partial y} - 5u = x.$$

6. Find the steady state temperature of a rod of length 10cm whose ends are kept at 30°C and 40°C respectively.

7. State the exponential form of Fourier integral theorem.

8. Find the Fourier sine transform of a function  $f(x) = e^{-2x} + 4e^{-3x}$ .

9. Find the  $Z$ -transform of  $(-1)^n$ .

10. State initial value theorem in  $Z$ -transforms.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the singular solution of  $z = px + qy + p^2 + pq + q^2$ . (8)

(ii) Solve  $(D^3 - 6D^2D' + 12DD'^2 - 8D'^3)z = e^{2x+y}$ . (8)

Or

(b) (i) Find the general solution of  $(z^2 - 2yz - y^2) \frac{\partial z}{\partial x} + x(y + z) \frac{\partial z}{\partial y} = x(y - z)$ . (8)

(ii) Solve  $(D^2 - 5DD' + 6D'^2)z = y \sin x$ . (8)

12. (a) (i) Find the Fourier series for the function  $f(x) = |x|$ ,  $-l < x < l$ . Hence find the value of  $1^{-2} + 3^{-2} + 5^{-2} + \dots$ . (8)

(ii) Find the half-range Fourier sine series for  $f(x) = x(\pi - x)$  in  $(0, \pi)$ , and hence show that  $1 - \frac{1}{3^3} + \frac{1}{5^3} - \dots = \frac{\pi^3}{32}$ . (8)

Or

(b) (i) Find the complex form of the Fourier series of a function  $f(x) = e^{-ax}$  in the interval  $(-l, l)$ . (8)

(ii) Find the half-range cosine series for  $f(x) = \begin{cases} x, & 0 < x < \pi/2 \\ \pi - x & \pi/2 < x < \pi \end{cases}$ .

Hence deduce the value  $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2}$ . (8)

13. (a) A string is stretched and fastened to two points  $l$  apart. Motion is started by displacing the string into the form of the curve  $y = x(l - x)$  and also by imparting a constant velocity ' $k$ ' to every point of the string in this position at time  $t = 0$ . Find the displacement function  $y(x, t)$ . (16)

Or

- (b) Find the steady state temperature distribution in a rectangular plate of sides ' $a$ ' and ' $b$ ' which is insulated on the lateral surface and three of whose edges  $x = 0$ ,  $x = a$ ,  $y = b$  are kept at zero temperature, if the temperature in the edge  $y = 0$  is given by  $3 \sin \frac{2\pi x}{a} + 2 \sin \frac{3\pi x}{a}$ . (16)

14. (a) (i) Find the Fourier transform of  $f(x) = \begin{cases} 1 - x^2, & -1 < x < 1 \\ 0 & \text{otherwise} \end{cases}$ . Hence

$$\text{evaluate } \int_0^{\infty} \left( \frac{\sin x - x \cos x}{x^3} \right) \cos \frac{x}{2} dx. \quad (10)$$

- (ii) Use Fourier cosine transforms method to evaluate

$$\int_0^{\infty} \frac{1}{(x^2 + 1)(x^2 + 4)} dx. \quad (6)$$

Or

- (b) (i) Find the Fourier transform of  $f(x) = \frac{\sin ax}{x}$  and hence find the

$$\text{value of } \int_{-\infty}^{\infty} \frac{\sin^2 ax}{x^2} dx, \text{ using Parseval's identity.} \quad (8)$$

- (ii) Find the Fourier cosine transform of  $e^{-ax}$ . Use it to find the Fourier transform of  $e^{-a|x|} \cos bx$ . (8)

15. (a) (i) Find the  $Z$ -transform of  $\frac{2n + 3}{(n + 1)(n + 2)}$ . (8)

- (ii) Solve, by using  $Z$ -transform, the equation  $y_{n+2} + 4y_{n+1} + 4y_n = n$ , given that  $y_0 = 0$  and  $y_1 = 1$ . (8)

Or

(b) (i) Find the  $Z$ -transform of  $f(n) * g(n)$ , where  
 $f(n) = \begin{cases} (1/3)^n, & n \geq 0 \\ (1/2)^{-n}, & n < 0 \end{cases}$  and  $g(n) = (1/2)^n U(n)$ . (8)

(ii) Find the inverse  $Z$ -transform of  $\frac{4z^3}{(2z-1)^2(z-1)}$ , by using the method of partial fractions. (8)

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