14. (a) (i) State and prove Cauchy's integral theorem. (8)

(ii) Evaluate the integral $\oint_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)} dz$, where C is the circle $|z| = 3$, using Cauchy's integral formula. (8)

Or

(b) (i) Find the Laurent's expansion of $f(z) = \frac{7z-2}{(z+1)z(z-2)}$ in the region $1 < z+1 < 3$. (8)

(ii) Evaluate $\int_{-\infty}^{\infty} \frac{x^2 dx}{(x^2+1)(x^2+4)}$, using Cauchy Residue integral. (8)

15. (a) (i) Solve the differential equation $(D^2 + D + 1)y = (1 - e^x)^2$. (8)

(ii) Using method of variation of parameters, solve the differential equation $\frac{d^2y}{dx^2} + 4y = \tan 2x$. (8)

Or

(b) (i) Solve the Legendre's linear equation

$$(1+x)^2 \frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = 2\sin[\log(1+x)].$$ (8)

(ii) Solve the simultaneous equations $\frac{dy}{dt} + 5x - 2y = t$, $\frac{dy}{dt} + 2x + y = 0$. (8)

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Question Paper Code : 30239

B.E./B.Tech. DEGREE EXAMINATIONS, APRIL/MAY 2023.

Third Semester

Electrical and Electronics Engineering

MA 3303 — PROBABILITY AND COMPLEX FUNCTIONS

(Regulations 2021)

Time : Three hours Maximum : 100 marks

Note : Statistical table to be permitted.

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. What are the axioms of probability?

2. Give the moment generating function of Binomial and Poisson distributions.

3. Define Central limit theorem.

4. Define covariance of random variables X and Y. Define their independence using covariance.

5. What are the necessary conditions for a function to be analytic?

6. What is conformal mapping?

7. Show that $\oint_c \frac{dz}{z-a} = 2\pi i$, where '$a$' is any point within simple closed curve 'C'.

8. Find the nature of singularity of $f(z) = \frac{z - \sin z}{z^2}$.

9. Solve the differential equation $(D^2 + 5D + 6)y = 0$.

10. Why does the method of undetermined coefficients fail when trial solution is assumed as $X = \tan x$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) A manufacturer of tablets receives its LED screens from three different suppliers B_1, B_2 and B_3. The probability that any one LED screen received by the plant comes from these three suppliers are, 0.60, 0.30 and 0.10 respectively. Suppose that 95% of the LED screens from B_1, 80% of those from B_2 and 65% of those from B_3 perform according to specifications. What is the probability that the LED screen received from any plant, performs according to specification. Also find the probability that LED screen working under specification has come from (1) Supplier B_1, (2) Supplier B_2, (3) Supplier B_3. (8)

(ii) Each sample of water has a 10% chance of containing a particular organic pollutant. Assume that the samples are independent with regard to the presence of the pollutant. Find the probability that in the next 18 samples, (8)

(1) Exactly 2 contain pollutant.

(2) Atleast four samples contain pollutant.

Or

(b) (i) The maximum attenuation occurring in the barcode scanner, changes from product to product. After collecting considerable data, the engineers decided to model the variation occurring as normal distribution with mean 10.1 dB and standard deviation 2.7 dB. For the next product what is the probability that its maximum attenuation is between 8.5 dB and 13.0 dB. Also what proportion of the products has maximum attenuation greater than 15.1 dB? (8)

(ii) In a large corporate network, the time interval between user log-ons to the system can be modelled as exponential distribution with mean 1/25 log-ons per hour. What is the probability that (8)

(1) There are no log-ons in an interval of six minutes?

(2) The time until next log-on is between two and three minutes?

(3) Determine the interval time such that the probability that no log-on occurs in the interval is 0.90.

12. (a) Determine the value of c that makes the function $f(x,y) = c(x+y)$, a joint probability mass function over the nine points with $x = 1,2,3$ and $y = 1,2,3$. (16)

(i) What is the marginal probability distribution of random variable X and Y?

(ii) What is the conditional probability distribution of Y given X = 1?

(iii) Find $P(X < 2, Y < 2)$, $P(X = 1, Y < 2)$

(iv) Find covariance (X, Y). Are X and Y independent?

Or

(b) An engineer conducts an experiment with the purpose of showing that adding a new component to the existing metal alloy increases the cooling rate. Let X denote the percentage of the new component present in the metal. Let Y denote the cooling rate, during a heat treatment stage in degree Fahrenheit per hour. The observed data are

X	0	1	2	2	4	4	5	6
Y	25	20	30	40	45	50	60	50

Fit a simple linear regression equation, for the given data. Estimate the value of cooling rate when new component percentage is 5.5%, using the fitted equation. Calculate the residuals and the error sum of squares of the fitted line. Estimate the correlation coefficient of the given data. (16)

13. (a) (i) If $f(z)$ is analytic function with constant modulus, show that $f(z)$ is constant. (8)

(ii) If $\omega = \phi + i\psi$ represents the complex potential for an electric field and $\psi = x^2 - y^2 + \frac{x}{x^2+y^2}$ determine the function. (8)

Or

(b) (i) Find the bilinear transformation which maps the points $z = 1, i, -1$ onto the points $w = i, 0, -i$ Hence find the image of $|z| < 1$. (8)

(ii) Interpret the transformation of points from z-plane to w-plane by (1) Translation, (2) Rotation, (3) Inversion. (8)