Reg. No. : [ ]

Question Paper Code : 60800

M.E./M.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2023.

First Semester

Aeronautical Engineering

MA 4153 – ADVANCED MATHEMATICAL METHODS

(Common to : M.E. Aerospace Technology/M.E. Soil Mechanics and Foundation Engineering/M.E. Structural Engineering)

(Regulations 2021)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Find the Laplace Transform of $f(t)=\begin{cases}\sin t, & 0<t<\pi \\ 0, & t>\pi\end{cases}$.

2. Find the inverse Laplace Transform of $\frac{s-3}{s^2+4s+13}$.

3. If $f(t)$ and $F(s)$ are Fourier transform pairs, then find $F\{e^{iat}f(t), s\}$ and $F\{f(t-a), s\}$.

4. State Convolution theorem of Fourier Transforms.

5. Define the Euler equation.

6. Write down the differential equation of the extremal of $\int_0^1 y'^2 dx$, $y(0)=0$, $y(1)=1$ is extremum, subject to the condition $\int_0^1 y\,dx = 2$.

7. Define Jacobian determinant.

8. Define bilinear transformation and its determinant.

9. Define Contravariant and Covariant vectors.

10. For an invariant function f, show that $curl(grad f)=0$.

PART B — (5 × 13 = 65 marks)

11. (a) Find $L^{-1}\left[\frac{s}{(s^2+1)(s^2+4)(s^2+p)}\right]$ using the convolution formula. (13)

Or

(b) Solve the one-dimensional heat equation $\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}$, $x>0$, $t>0$ with boundary conditions $u(x,0)=0$, $u(0,t)=\delta(t)$, $u(\infty,t)=0$ using Laplace transform. (13)

12. (a) Compute the displacement $u(x,t)$ of an infinite string using the method of Fourier transform given that the string is initially at rest and that the initial displacement is $f(x)$, $-\infty<x<\infty$. (13)

Or

(b) Using the method of integral transform solve the following potential problem in the semi-infinite strip described by PDE: $u_{xx}+u_{yy}=0$, $0<x<\infty$, $0<y<a$, Subject to the boundary conditions, $u(x,0)=f(x)$, $u(x,a)=0$, $u(x,y)=0$, $0<y<a$, $0<x<\infty$ and $\frac{\partial u}{\partial x}\to 0$ as $x\to\infty$. (13)

13. (a) Find the extremal of the functional $I=\int_{t_1}^{t_2}\left(x'y'+2x^2+2y^2\right)dt$; $x'=\frac{dx}{dt'}$, $y'=\frac{dy}{dt}$, subject to the conditions; at $t_1=0$, $x=0$, $y=0$ but the end t^2 moves on the plane $t=t_2$. (13)

Or

(b) Solve the boundary value problem $x^2y''+xy'-y=2x^2$, $y'=\frac{dy}{dx}$, $y''=\frac{d^2y}{dx^2}$ subject to the conditions $y(0)=0$ and $y(1)=1$ by using Ritz method. (13)

14. (a) Find the bilinear transformation (BLT) which maps the points $z=1$, $i,-1$ onto the points $w=i,0,-i$. Also, find the image of $|z|<1$. Hence, find the invariant points of this BLT, transformation. (13)

Or

(b) Find the electrical potential $Q(x, y)$ produced by two charged half-planes that are perpendicular to the z-plane and pass through the rays $x<-1, y=0$ and $x>1, y=0$ where the planes are kept at the fixed potentials $Q(x,0)=-300$ for $x<-1$ and $Q(x,0)=300$ for $x>1$. (13)

15. (a) A covariant vector has components $xy, 2y - z^2, xz$ in rectangular Cartesian coordinates. Determine its components in spherical polar coordinates.

Or

(b) If the components A_1, A_2, A_3 of a vector in cylindrical coordinates P, Q, Z are $P, Z\sin\phi, e^{\phi}\cos Z$ then find $div A_i$. (13)

PART C — (1 × 15 = 15 marks)

16. (a) (i) Find the Fourier Transform of $f(x) = e^{-a|x|}, -\infty < x < \infty$. (7)

(ii) Using the Laplace transform method, solve $u_{tt} = u_{xx}$, $0 < x < 1, t > 0$, with boundary conditions $u(0, t) = u(1, t) = 0, t > 0$ and initial conditions $u(x, 0) = \sin \pi x$, $u_t(x, 0) = -\sin \pi x$, $0 < x < 1$. (8)

Or

(b) Calculate the Christoffel symbols $[i\ j\ k]$ and $\left\{ \begin{matrix} i \\ j\ k \end{matrix} \right\}$ corresponding to the metric $ds^2 = (dx')^2 + (x')^2 (dx^2)^2 + (x' \sin x^2)^2 (dx^3)^3$. (15)