Reg. No. : [| | | | | | | | | | | |]

Question Paper Code : 21282

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2023.

Third Semester

Computer Science and Engineering

MA 3354 – DISCRETE MATHEMATICS

(Common to: Computer Science and Engineering (Artificial Intelligence and Machine Learning)/Computer Science and Engineering (Cyber Security)/Computer and Communication Engineering/Artificial Intelligence and Data Science/Computer Science and Business Systems and Information Technology)

(Regulations 2021)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Write the symbolic form of the statement "The automated reply cannot be sent when the file system is full".

2. Check whether the conclusion C : q follows logically from the premises $H_1 : p \to q$ and $H_2 : q$ using truth table technique.

3. How many different bit strings are there of length 9?

4. Determine n if $P(n, 2) = 72$.

5. Define a complete graph.

6. When is a graph called an Eulerian graph?

7. Identify the left cosets of {[0], [3]} in the group $(Z_6, +_6)$.

8. Given $G = \{1, -1, i, -i\}$ be a group and $H = \{-1, 1\}$ be a subgroup of G. Find the index of H in G.

9. Draw the Hasse diagram of $(S_{24}, /)$ where S_{24} is the set of all positive divisors of 24 and / is division.

10. Prove that $(a')' = a$, for all $a \in B$ where B is a Boolean Algebra.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Construct the truth table for the following statement (8)

$\neg(p \vee (q \wedge r)) \leftrightarrow ((p \vee q) \wedge (p \vee r))$.

(ii) "If there was a ball game, then travelling was difficult. If they arrived on time, then travelling was not difficult. They arrived on time. Therefore, there was no ball game". Show that these statements constitute a valid argument. (8)

Or

(b) (i) Construct the principal disjunctive normal form of $p \rightarrow [(p \rightarrow q) \wedge \neg(\neg q \vee \neg p)]$. (8)

(ii) Use the indirect method to prove that the conclusion $\exists z Q(z)$ from the premises $\forall x(P(x) \rightarrow Q(x))$ and $\exists y P(y)$. (8)

12. (a) (i) Suppose a department consists of eight men and women. In how many ways can we select a committee of (8)

(1) Three men and four women?

(2) Four persons that has at least one woman?

(3) Four persons that has at most one man?

(4) Four persons that has persons of both gender?

(ii) Use generating functions to solve the recurrence relation (8)

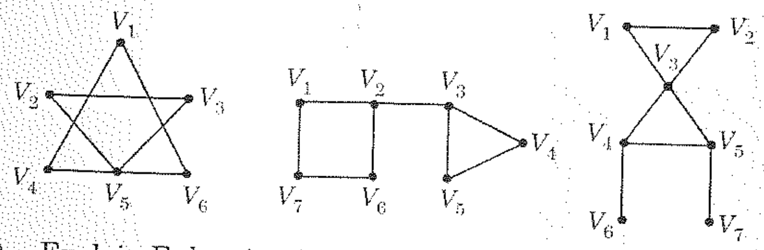
$a_n = 3a_{n-1} + 1; n \geq 1$ *with* $a_0 = 1$.

Or

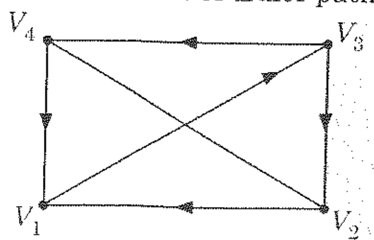
(b) (i) Use mathematical induction to prove that $(3^n + 7^n - 2)$ is divisible by 8, for $n \geq 1$. (8)

(ii) Solve the recurrence relation $a_n = 2a_{n-1} + 2^n, a_0 = 2$. (8)

13. (a) (i) Define a connected graph. Which of the given graphs are connected? (8)



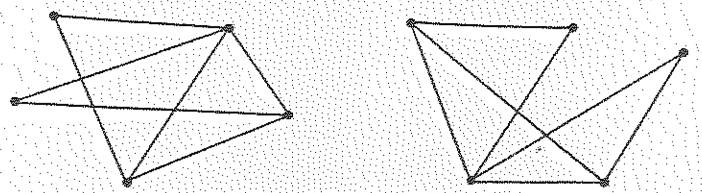
(ii) Explain Euler circuit and Euler path. Determine whether the given graph G has an Euler circuit or Euler path. (8)



Or

(b) (i) Prove that the maximum number of edges in a simple disconnected graph G with 'n' vertices and 'k' components is $\frac{(n-k)(n-k+1)}{2}$. (8)

(ii) Examine whether the following pair of graphs are isomorphic. If isomorphic, label the vertices of the two graphs to show that their adjacency matrices are the same. (8)



14. (a) (i) State and prove Lagrange's theorem. (8)

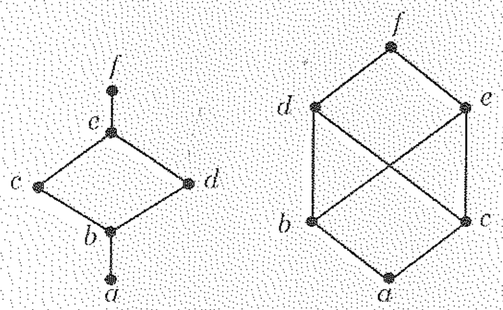
(ii) If $(G, *)$ is a group, prove that $(a*b)^{-1} = b^{-1} * a^{-1}$ for all $a, b \in G$. (8)

Or

(b) Show that $(Z, \oplus, \odot)$ is a commutative ring with identity, where the operations $\oplus$ and $\odot$ are defined, for any $a, b \in Z$ as $a \oplus b = a + b - 1$ and $a \odot b = a + b - ab$. (16)

15. (a) (i) Let $(L, \leq)$ be a lattice and $a, b, c, d \in L$. If $a \leq c$ and $b \leq d$, Then prove that (1) $a \vee b \leq c \vee d$ (2) $a \wedge b \leq c \wedge d$. (8)

(ii) Verify whether the poset represented by the each of the Hasse diagrams are lattices. (8)



Or

(b) (i) State and prove De Morgan's laws in any Boolean Algebra. (8)

(ii) If $a, b \in S = \{1, 2, 4, 8, 16\}$ and $a \vee b = l.c.m.$ of $\{a, b\}$, $a \wedge b$ = $g.c.d$ *of* $\{a, b\}$ and $a' = 16/a$, then show that $\{S, \vee, \wedge, ', 1, 16\}$ is not a Boolean algebra. (8)
