Reg. No. : | | | | | | | | | | | | |

Question Paper Code : 70858

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2023.

Fourth Semester

Computer and Communication Engineering

MA 8451 – PROBABILITY AND RANDOM PROCESSES

(Common to : Electronics and Communication Engineering/Electronics and Telecommunication Engineering)

(Regulations 2017)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. State the axioms of probability.

2. If $P(A)=\frac{1}{3}$, $P(B)=\frac{3}{4}$ and $P(A\cup B)=\frac{11}{12}$. Find $P(A/B)$ and $P(B/A)$.

3. The joint probability density function of (X,Y) is given by $f(x,\ y)=e^{-(x+y)}$, $0\leq x,\ y<\infty$. Are the random variables X and Y independent? Why?

4. State the central limit theorem

5. Write the classification of random process.

6. What is meant by evolutionary process?

7. The autocorrelation function for a stationary ergodic process with no periodic components is $R_{XX}(\tau)=25+\frac{4}{1+6\tau^2}$. Find the mean of the process $\{X(t)\}$.

8. State any two properties of the power spectral density function.

9. When is the system $Y(t)=\int_{-\infty}^{\infty} h(u)X(t-u)du$ said to be stable?

10. The short-time moving average of a process $\{X(t)\}$ is defined as $Y(t)=\frac{1}{T}\int_{t-T}^{t} X(s)ds$. Prove that $X(t)$ and $Y(t)$ are related by means of a convolution type integral.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Customers are used to evaluate preliminary product designs. In the past, 95% of highly successful products received good reviews, 60% of moderately successful products received good reviews, and 10% of poor products received good reviews. In addition, 40% of products have been highly successful, 35% have been moderately successful and 25% have been poor products.

(1) What is the probability of a new design attains a good review?

(2) If a new design attains a good review, what is the probability that it will be a highly successful product? (8)

(ii) A random variable X has the following probability distribution. (8)

x	:	–2	–1	0	1	2	3
$p(x)$	:	0.1	K	0.2	2K	0.3	3K

Evaluate the following (1) K (2) $P(-2<X<2)$ (3) the cdf of X (4) mean of X.

Or

(b) (i) Obtain the MGF of the Binomial distribution and hence find its mean and variance. (8)

(ii) The mileage which car owners get with a certain kind of radial tire is a random variable having an exponential distribution with mean 40,000 km. Find the probabilities that one of these tires will last (1) at least 20000 km and (2) at most 30,000 km. (8)

12. (a) (i) The joint probability mass function of (X, Y) is given by $f(x, y)=C(2x+3y)$, $x=0,1,2$; $y=1,2,3$. Find the marginal probability distributions of X and Y. (8)

(ii) Compute the co-efficient of correlation between X and Y, using the following data : (8)

X	:	1	2	3	4	5
Y	:	2	5	9	13	14

Or

(b) (i) The joint probability density function of a two-dimensional random variable (X, Y) is given by (8)

$f(x,y) = xy^2 + \frac{x^2}{8}, 0 \le x \le 2, 0 \le y \le 1$. Compute

(1) $P\left(X > 1, Y < \frac{1}{2}\right)$

(2) $P\left(Y < \frac{1}{2}\right)$

(3) $P(X < Y)$

(ii) If X and Y are independent RVs each following $N(0, 2)$, find the pdf of $Z = 2X + 3Y$. (8)

13. (a) (i) The probability distribution of the process $\{X(t)\}$ is given by (8)

$$P(X(t) = n) = \begin{cases} \frac{(at)^{n-1}}{(1+at)^{n+1}}, & n = 1,2,3,... \\ \frac{at}{1+at}, & n = 0 \end{cases}$$

. Show that it is not stationary.

(ii) The transition probability matrix of a Markov chain $\{X_n\}$, $n = 1,2,3...$, having 3 states 1, 2 and 3 is $P = \begin{bmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{bmatrix}$ and the initial distribution is $P^{(0)}$=(0.7, 0.2, 0.1). Find $P(X_2 = 3)$ and $P(X_2 = 3, X_1 = 3, X_0 = 2)$. (8)

Or

(b) (i) Show that the random process $X(t) = A\cos(\omega t + \theta)$ is wide sense stationary if A and ω are constant and θ is uniformly distributed random variable in $(0, 2\pi)$. (8)

(ii) Prove that the difference of two independent Poisson processes is not a Poisson process. (8)

14. (a) (i) If $\{X(t)\}$ is a wide-sense stationary process with autocorrelation $R(\tau) = Ae^{-\alpha|\tau|}$, determine the second-order moment of the RV $X(8) - X(5)$. (8)

(ii) If the power spectral density of a WSS process is given by $S(\omega) = \begin{cases} \frac{b}{a}(a - |\omega|), & |\omega| \le a \\ 0, & |\omega| > a \end{cases}$. Find the autocorrelation function of the process. (8)

Or

(b) (i) The autocorrelation function of the random telegraph signal process is given by $R(\tau) = a^2 e^{-2\gamma|\tau|}$. Determine the power density spectrum of the random telegraph signal. (8)

(ii) If the WSS process $\{X(t)\}$ is given by $X(t) = 10\cos(100t + \theta)$, where θ is uniformly distributed over $(-\pi, \pi)$, prove that $\{X(t)\}$ is correlation ergodic. (8)

15. (a) State and prove Wiener-khinchine theorem for a linear system with random inputs. (16)

Or

(b) (i) The process $\{X(t)\}$ is normal with $\mu_t = 0$ and $R_X(\tau) = 4e^{-3|\tau|}$. Find a memory less system $g(x)$ such that the first order density $f_Y(y)$ of the resulting output $Y(t) = g[X(t)]$ is uniform in the interval (3, 6). (8)

(ii) Consider a white Gaussian noise of zero mean and power spectral density $N_0/2$ applied to a low pass RC filter whose transfer function is $H(f) = \frac{1}{1 + i2\pi fRC}$. Find the auto correlation function of the output random process. (8)