Reg. No. : | | | | | | | | | | | | |

Question Paper Code : 21279

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2023

Third Semester

Civil Engineering

MA 3351 – TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

(Common to all branches)

(Regulations 2021)

Time : Three hours   Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Obtain the partial differential equations by eliminating arbitrary constants a and b from $(x+a)^2+(y-b)^2=z$.

2. Solve $(D^2-4DD'+3D'^2)z=0$.

3. State the condition for a function $f(x)$ to be expressed as a Fourier series.

4. If $f(x)=x^2+x$ is a expresses as a Fourier series in the interval $(-2,2)$ to which value this series converges at $x=2$.

5. Classify the partial differential equation $u_{xx}-y^4u_{yy}=2y^3u_y$.

6. Write down the various solutions of one dimensional wave equation.

7. State the convolution theorem of Fourier Transform.

8. Prove that $F[f(x-a)]=e^{isa}F(s)$.

9. Find the Z transform of 1.

10. State initial and final value theorem of Z transforms.

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PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the general solution of $x(y^2 - z^2)p + y(z^2 - x^2)q = z(x^2 - y^2)$. (8)

(ii) Solve the partial differential equation $[D^2 + 3DD' + 2D'^2]z = x + y$. (8)

Or

(b) (i) Obtain the singular solution of the partial differential equation. $(pq - p - q)(z - px - qy) = pq$. (8)

(ii) Solve the partial differential equation $[D^2 + 2DD' + D'^2]z = x^2y + e^{x-y}$. (8)

12. (a) (i) Expand the Fourier series for $f(x) = x(2\pi - x)$ in $(0,2\pi)$ and hence deduce that the sum of $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + ... = \frac{\pi^2}{6}$. (10)

(ii) Expand the function $f(x) = x,\ 0 < x < \pi$ in the Fourier sine series. (6)

Or

(b) (i) Obtain the function $f(x) = \sin x,\ 0 < x < \pi$ in Fourier cosine series. (8)

(ii) Determine the first two harmonic of the Fourier series for the following values (8)

x:	0	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$
y:	1.98	1.30	1.05	1.30	–0.88	–0.25

13. (a) A string is tightly stretched and its ends are fastened at the two points $x = 0$ and $x = 2l$. The mid-point of the string is displaced transversely through a small distance b and the string is released from rest in that position. Obtain an expression for the displacement of the string at any subsequent time.

Or

(b) The ends A and B of a rod 30 cm long have their temperature kept at 20°C. and 80°C respectively until steady state condition prevail. The temperature at each end is then suddenly reduced to 60°C and that of A is raised 40°C. Find the temperature distribution in the rod after time t.

14. (a) (i) Find the Fourier sine and cosine transform of e^{-ax} and hence deduce the inversion formula. (8)

(ii) Find the Fourier integral of the function $f(t) = \begin{cases} e^{at}, & t < 0 \\ e^{-at}, & t > 0 \end{cases}$. (8)

Or

(b) Obtain the Fourier transform of $f(x) = \begin{cases} 1 - |x|, & if\ |x| < 1 \\ 0, & if\ |x| > 1 \end{cases}$ Hence deduce that $\int_0^\infty \left(\frac{\sin t}{t}\right)^2 dt = \frac{\pi}{2}$, Using Parseval's identity find the value of $\int_0^\infty \left(\frac{\sin t}{t}\right)^4 dt$.

15. (a) (i) Find the Z – transforms of $\cos\left(\frac{n\pi}{2}\right)$ and $\frac{2n+3}{(n+1)(n+2)}$. (8)

(ii) Obtain the inverse Z–transforms of $\frac{z^2+2z}{z^2+2z+5}$. (8)

Or

(b) (i) Find the inverse Z–transforms of $\frac{2z^2+3z}{(z+2)(z-4)}$. (8)

(ii) Solve the difference equation $y_{n+2} + 4y_{n+1} + 3y_n = 3^n$ with $y_0 = 0$, $y_1 = 1$ by using Z transforms. (8)

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