Reg. No. : [| | | | | | | | | | | |]

Question Paper Code : 21277

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2023.

Third Semester

Electrical and Electronics Engineering

MA 3303 — PROBABILITY AND COMPLEX FUNCTIONS

(Regulations 2021)

Time : Three hours　　　　Maximum : 100 marks

(Use the statistical tables is permitted)

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. If $P(C) = 0.65$, $P(D) = 0.40$ and $P(C \cap D) = 0.24$, Are the events C and D Independent?

2. Check whether the function $f(x) = \frac{x-2}{2}$ for $x = 1, 2, 3, 4$. Can serve as probability distributions.

3. If X_1 has mean 4 while X_2 has mean –2, find $E(2X_1 + X_2 - 5)$.

4. State the central limit theorem.

5. Let $f(z) = z^2 + 3z$. Find u and v and calculate the value of f at $z = 1 + 3i$.

6. Define conformal mapping.

7. Find the residue of $f(z) = \frac{\sin z}{z \cos z}$ at its pole inside the circle $|z| = 1$.

8. What type of singularities have the following function?

   (a) $f(z) = \frac{1}{1 - e^z}$　　　　(b) $f(z) = \frac{e^{\frac{1}{z}}}{z^2}$

9. Are the function $e^{-x} \cos wx$ and $e^{-x} \sin wx$, $w \neq 0$ linearly independent? Justify.

10. Does the super position principal hold for non-homogeneous linear ODE? Justify.

PART B — (5 × 16 = 80 marks)

11. (a) (i) A first step towards identifying spam is to create a list of words that are more likely to appear in spam than in normal messages. For instance, words like buy or the brand name of an enhancement drug are more likely to occur in spam messages than in normal messages. Suppose a specified list of words is available and that your data base of 5000 messages contains 1700 that are spam. Among the spam messages, 1343 contain words in the list. Of the 3300 normal messages, only 297 contain words in the list. Obtain the probability that a message is spam given that the message contains words in the list. (8)

(ii) Verify that the functions $f(x) = \frac{2x+1}{25}, x = 0, 1, 2, 3, 4$ are probability mass functions, and determine the following: (8)

(1) $P(X = 4)$ (2) $P(X \leq 2)$

(3) $P(2 \leq X < 4)$ (4) $P(X \geq 2)$

Or

(b) (i) The random variable X has a binomial distribution with $n = 10$ and $p = 0.1$. Determine the following probabilities. (8)

(1) $P(X \leq 2)$ (2) $P(X > 8)$

(3) $P(X = 4)$ (4) $P(5 \leq X \leq 7)$

(ii) Suppose X has an exponential distribution with mean equal to 10. Determine the following: (8)

(1) $P(X > 10)$ (2) $P(X < 30)$

(3) Find the value of x such that $P(X < x) = 0.95$

12. (a) Determine the value of c that makes the function $f(x, y) = c(x + y)$ a joint probability mass function over the nine points with $X = 1, 2, 3$ and $Y = 1, 2, 3$. Determine the following: (16)

(i) $P(X = 5, Y < 4)$

(ii) $P(X = 1)$

(iii) $P(Y = 4)$

(iv) Marginal probability distribution of the random variable X

(v) Find correlation and covariance

Or

(b) (i) Determine the value of c such that the function $f(x,y) = cxy$, for $0 < x < 3$ and $0 < y < 3$ satisfies the properties of a joint probability density function. Determine the following: (8)

(1) $P(X < 2, Y < 3)$ (2) $P(X < 2.5)$

(3) $E(X), E(Y)$ (4) $Var(X)$ and $Var(Y)$

(5) Conditional probability distribution of Y given that $X = 1.5$

(ii) Find a least square straight line for the following data (8)

X	1	2	3	4	5	6
Y	6	4	3	5	4	2

13. (a) (i) State and prove Cauchy-Riemann equations for analytic function. (8)

(ii) Verify that $u(x,y) = x^2 - y^2 - y$ is harmonic in the whole complex plane and find a harmonic conjugate function v of u. (8)

Or

(b) (i) Show that $w = \frac{i-z}{i+z}$, maps the real axis z-plane into the circle $|w| = 1$. (8)

(ii) Find the bilinear transformation which maps the points $z = -1, i, 1$ onto the points $w = 0, i, \infty$ respectively. (8)

14. (a) State and prove Cauchy's integral formula. Use this formula to evaluate $\int_c \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)} dz$ where C is the circle $|z| = 3$. (8+8)

Or

(b) (i) Find the Laurent's series expansion of $f(z) = \frac{7z-2}{z(z+1)(z-2)}$ in the region $1 < z+1 < 3$. (8)

(ii) Evaluate the real integral $\int_{-\infty}^{\infty} \frac{x^2 dx}{(x^2+1)(x^2+4)}$ by using Cauchy's residue theorem. (8)

15. (a) (i) Find the general solution of the Euler-Cauchy equations $x^2y'' + xy' + 16y = 0$. (6)

(ii) Solve by method of variation of parameters $y'' - 2y' + y = e^x \log x$. (10)

Or

(b) (i) Solve the simultaneous equations $\frac{dx}{dt} + 2y + \sin t = 0, \frac{dy}{dt} - 2x - \cos t = 0$ Given that $x = 0$ and $y = 1$ when $t = 0$. (8)

(ii) Solve by the method of undetermined coefficients $(D^2 + 1)y = \sin x$. (8)
