b) i) In a factory cafeteria, the customers have to pass through three counters. The customers buy coupons at the first counter, select and collect the snacks at the second counter and collect tea at the third. The server at each counter takes, on an average, 1.5 minutes, although the distribution of service time is approximately Poisson at an average rate of 6 per hour. Calculate

1) The average time a customer spends waiting in the cafeteria.

2) The average time of getting the service.

3) The most probable time in getting the service. **(10)**

ii) Consider a queueing system where a firm is engaged in both shipping and receiving activities. The management is always interested in improving the efficiency of new innovation in loading and unloading procedures. The arrival distribution of trucks is found to be Poisson with arrival rate of 3 trucks per hour. The service time distributions is exponential with unloading rate of 4 trucks per hour. Determine expected waiting time of the truck in the queue and what reductions in waiting time are possible if loading and unloading is standardized ? **(6)**

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13.11.17 AN

Reg. No. :

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B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2017

Fourth Semester

Mechanical Engineering (Sandwich)

MA 6453 – PROBABILITY AND QUEUEING THEORY

(Common to Computer Science and Engineering/Information Technology)

(Regulations 2013)

Time : Three Hours Maximum : 100 Marks

Answer ALL questions

PART – A (10×2=20 Marks)

1. A test engineer discovered that the cumulative distribution function of the lifetime of an equipment (in years) is given by $F_X(x) = 1 - e^{-\frac{x}{5}}$, $x \geq 0$. What is the expected lifetime of the equipment ?

2. If X is a normal random variable with mean 3 and variance 9, find the probability that X his between 2 and 5.

3. Let the joint probability density function of random variables X and Y be given by $f(x, y) = 8xy$, $0 \leq y \leq x \leq 1$. Calculate the marginal probability density function of X.

4. If X and Y are random variables having the joint density function $f(x, y) = \frac{1}{8}(6 - x - y)$, $0 < x < 2$, $2 < y < 4$, find $P[x + y < 3]$.

5. Define Markov Chain and one-step transition probability.

6. State any two properties of a Poisson process.

7. Arrivals at telephone booth are considered to be Poisson with an average time of 10 minutes between one arrival and the next. The length of phone calls is assumed to be distributed exponentially with a mean of 3 minutes. What is the average length of the queue that forms from time to time ?

8. Define the following terms : Balking, Reneging and Jockeying.

9. Find the length of the queue for an M/G/1 model if $\lambda = 5$, $\mu = 6$ and $\sigma = 1/20$.

10. Define series queue model.

PART – B (5×16=80 Marks)

11. a) i) Let X be a continuous random variable with the probability density function $f(x) = 1/4,\ 2 \le x \le 6$. Find the expected value and variance of X. (6)

ii) Find the moment generating function of a normal distribution and hence find its mean and variance. (10)

(OR)

b) i) Messages arrive at a switchboard in a Poisson manner at an average rate of six per hour. Find the probability that atleast three messages arrive within one hour. (6)

ii) For a Gamma random variable X with parameters $(K, \lambda)$, derive the moment generating function and hence obtain its mean and variance. (10)

12. a) Two random variables X and Y have the following joint probability density function $f(x, y) = xe^{-x(y+1)}$, $x \ge 0$, $y \ge 0$. Determine the conditional probability density function of X given Y and the conditional probability density function of Y given X. (16)

(OR)

b) If X and Y are two independent random variables each normally distributed with mean 0 and variance $\sigma^2$, then find the joint probability density function of $R = \sqrt{x^2 + y^2}$ and $\theta = \tan^{-1}(y/x)$ and hence find the probability density function of $\theta$. (16)

13. a) i) A random process $\{x(t)\}$ has the probability distribution

$$P[X(t) = x] = \begin{cases} \dfrac{(at)^{x-1}}{(1+at)^{x+1}}, & x = 1,2,3\ldots \\ \dfrac{at}{1+at}, & x = 0 \end{cases}$$

Show that the process is non-stationary. (10)



ii) If $\{X(t)\}$ is a Poisson process with parameter $\lambda t$, then prove that $P[X(t_1) = x/X(t_2) = n] = nc_x p^x q^{n-x}$ where $p = \frac{t_1}{t_2}$, $q = 1 - p$. (6)

(OR)

b) i) Suppose the arrival of calls at a switch board is modelled as a Poisson process with the rate of calls per minute being $\lambda = 0.1$. What is the probability that the number of calls arriving in a 10 minutes interval is less than 3 ? (6)

ii) Consider a Markov Chain with 3 states and transition probability matrix $P = \begin{pmatrix} 0.6 & 0.2 & 0.2 \\ 0.1 & 0.8 & 0.1 \\ 0.6 & 0 & 0.4 \end{pmatrix}$. Find the stationary probabilities of the chain. (10)

14. a) i) Consider a single server queueing system with Poisson input and exponential service times. Suppose the mean arrival rate is 3 calling limits per units, the expected service time is 0.25 hour and the maximum permissible calling units in the system is two. Calculate the expected number in the system. (8)

ii) Show that for a single service station, Poisson arrivals and exponential service time, the probability that exactly n calling units in the queueing system is $p_n = (1 - e)e^n$, $n \ge 0$, where e is the traffic intensity. Also, find the expected number of units in the system. (8)

(OR)

b) Let there be an automobile inspection situation with three inspection stalls. Assume that cars wait in such a way that when a stall becomes vacant, the car at the head of the line pulls up to it. The station can accommodate almost four cars waiting at one time. The arrival pattern is Poisson with a mean of one car every minute during the peak hours. The service time is exponential with a mean of 6 minutes. Find the average number of customers in the system during the peak hours, the average waiting time and the average number per hour that cannot enter the station because of full capacity. (16)

15. a) i) Derive Pollaczek-Khinchine formula for M/G/1 queueing system. (10)

ii) In a big factory, there are a large number of operating machines and sequential repair shops which do the service of the damaged machines exponentially with respective 1/hour and 2/hour. If the cumulative failure rate of all the machines in the factory is 0.5/hour, find (1) the probability that both repair shops are idle. (2) the average number of machines in the service. (6)

(OR)