15. (a) In a network of 3 service stations 1,2,3 customers arrive at 1,2,3 from outside, in accordance with Poisson process having rates 5, 10, 15 respectively. The service times at the 3 stations are exponential with respective rates 10, 50, 100. A customer completing service at station 1 is equally likely to go to station 2 or go to station 3 or leave the system. A customer departing from service at station 2 always goes to station 3. A departure from service at station 3. 1 is equally likely to go to station 2 or leave the system. What is the average number of customers in the system? And what is the average time a customer spends in the system? (16)

Or

(b) Consider a queuing system where arrivals are according to a Poisson distribution with mean 5 per Hour. Find the expected waiting time in the system. If the service time distribution is (i) Uniform between: $t = 5$ minutes and $t = 15$ minutes (ii) Normal with mean 3 minutes and standard deviation 2 minutes. (16)

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Question Paper Code : 72073

B.E./B.Tech. DEGREE EXAMINATION, APRIL/MAY 2017.

Fourth Semester

Mechanical Engineering (Sandwich)

MA 6453 — PROBABILITY AND QUEUEING THEORY

(Common to Computer Science and Engineering/Information Technology)

(Regulations 2013)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. If the probability density function of a random variable X is $f(x) = \frac{1}{4}$ in $-2 < x < 2$ find $P(|X| > 1)$.

2. If X is a geometric variate, taking values 1, 2, 3, ...∞, find P(X is odd).

3. Define conditional distribution for two- dimensional discrete and continuous random variables.

4. If $X = R\cos\phi$ and $Y = R\sin\phi$, how are the joint probability density function of (X, Y) and (R, ϕ) are related?

5. Define a kth order stationary process. When will it become a strict sense stationary process?

6. State Chapman-Kolmogorov theorem.

7. State Little's formula for the queuing model $(M/M/S):(\infty/FIFO)$.

8. What are the values of P_0 and P_n for the queuing model (M/M/1): (K/FIFO) when $\lambda = \mu$

9. State Pollaczek - Khintchine formula for (M/G/1) queuing model.

10. Write down the traffic equation for open Jackson network.

PART B — (5 × 16 = 80 marks)

11. (a) (i) The density function of a continuous random variable X is given by

$$f(x)=\begin{cases} ax, & 0\le x\le 1\\ a, & 1\le x\le 2\\ 3(a-x), & 2\le x\le 3\\ 0 & \text{otherwise}\end{cases}$$

(1) Find the value of 'a' (2) CDF of X. (8)

(ii) The probability of a man hitting a target is 1/4. If he fires 7 times, what is the probability of his hitting the target at least twice? And how many times must he fire so that the probability of his hitting the target at least once is greater than 2/3? (8)

Or

(b) (i) Find the MGF of the random variable X having the probability density function

$$f(x)=\begin{cases} \frac{x}{4}e^{-x/2} & x>0\\ 0 & \text{otherwise}\end{cases}$$. Also find the first four moments about the origin. (8)

(ii) In a normal population with mean 15 and standard deviation 3.5, it is found that 647 observations exceed 16.25. What is the total number of observations in the population? (8)

12. (a) (i) The joint probability mass function of (X, Y) is given by $p(x, y) = k\,(2x+3y)\, x = 0, 1, 2;\ \ y = 1, 2, 3$. Find the value of k and find all the marginal probability distributions. (6)

(ii) The probability density function of (X, Y) is given by $f(x, y)=\begin{cases} 8xy, & 0<x<y<1\\ 0 & otherwise\end{cases}$. Find $P\left(X<\frac{1}{2}\cap y<\frac{1}{4}\right)$. Are X and Y independent? Justify your answer. (10)

Or

(b) (i) If the joint pdf of (X, Y) is given by $f(x, y)=e^{-(x+y)}\ \ x>0,\ y>0$, prove that X and Y are uncorrelated. (8)

(ii) If X and Y follows an exponential distribution with parameter 2 and 3 respectively and are independent, find the probability density functions of $U = X + Y$. (8)

13. (a) (i) If the process $X(t) = P + Qt$ where P and Q are independent random variables with $E(P) = p$, $E(Q) = q$, $\text{var}(P) = \sigma_1^2$ and $\text{var}(Q) = \sigma_2^2$, find $E\{X(t)\}$ and $R(t_1, t_2)$. Is the process $\{X(t)\}$ stationary. (8)

(ii) On the average a submarine on patrol sights 6 enemy ships per hour. Assuming that the number of Ships sighted in a given length of time is a Poisson variate. Find the probability of sighting 6 ships in the next half-an-hour, 4 ships in the next 2 hours and at least 1 ship in the next 15 minutes. (8)

Or

(b) (i) Three boys A, B, and C are throwing a ball to each other. A always throws the ball to B and B always throws the ball to C, but C is just likely to throw the ball to B as to A. Show that the process is Markovian. Find the transition matrix and classify the states. (8)

(ii) Prove that the random process $X(t) = \cos(w_0\, t + \theta)$ where θ is uniformly distributed in the interval $(-\pi, \pi)$ is wide sense stationary. (8)

14. (a) (i) Obtain P_o and P_n for the birth and death process. (8)

(ii) Arrives at a telephone booth are considered to be Poisson distribution, with an average time of 10 minutes between one arrival and the next. The length of phone call assumed to be distributed exponentially with mean 3 minutes. What is the probability that a person arriving at the booth will have to wait? And what is the average length of the queues that form from time to time? (8)

Or

(b) There are three typists in an office. Each typist can type an average of 6 letters per hour. If letters arrive for being typed at the rate of 15 letters per hour. What is the probability of all the typists will be busy? What is the average number of letters waiting to be typed? What is the average time a letter has to be spent for waiting and for being typed? And what is the probability that a letter will take longer than 20 minutes waiting to be typed and being typed? (16)