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B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2019

Fourth Semester

Computer and Communication Engineering

MA8451 – PROBABILITY AND RANDOM PROCESSES

(Common to Electronics and Communication Engineering/Electronics and Telecommunication Engineering)

(Regulations 2017)

Time : Three Hours Maximum : 100 Marks

Answer ALL questions

PART – A (10×2=20 Marks)

1. Let A and B be two events such that P(A) = 0.5, P(B) = 0.3 and $P(A \cap B) = 0.15$. Compute P(B/A) and $P(\bar{A} \cap B)$.

2. The R.V. X has p.m.f. $P(X = x) = \begin{cases} \frac{c}{x}, & x = 1, 2, 3, \\ 0, & \text{otherwise} \end{cases}$.

   Obtain :

   i) The value of 'C' ii) $P(X \geq 2)$.

3. The R.V.s X and Y have joint p.d.f. $f(x, y) = \begin{cases} \frac{1}{15}, & 0 \leq x \leq 5, \\ & 0 \leq y \leq 3 \\ 0, & \text{otherwise} \end{cases}$. What is P(Y > X) ?

4. Prove that the correlation coefficient ρ_{XY} of the R.V.s X and Y takes value in the range – 1 and 1.

5. Define Markov process.

6. Let X(t) be a wide-sense stationary random process with E(X(t)) = 0 and $Y(t) = X(t) - X(t + \tau)$, $\tau > 0$. Compute E(Y(t)) and Var (Y(t)).

7. A stationary random process X(t) has an autocorrelation function $R_{XX}(\tau) = \frac{4\tau^2 + 6}{\tau^2 + 1}$. Find E(X(t)) and $E(X^2(t))$.

8. Determine which of the following functions are power spectrum, which are not ?

   i) $S_{XX}(\omega) = e^{-(\omega-2)^2}$ ii) $S_{XX}(\omega) = \frac{\cos^2\omega}{\omega^4 + 2\omega^2 + 1}$.

9. Define :

   i) Linear Time-Invariant System

   ii) Casual system.

10. A random process X(t) is the input to a linear system whose impulse response is $h(t) = 2e^{-t}$, $t \geq 0$. Given $R_{XX}(\tau) = e^{-3|\tau|}$, find the power spectral density of the output process y(t).

PART – B (5×16=80 Marks)

11. a) i) Companies B_1, B_2 and B_3 produce 30%, 45% and 25% of the cars respectively. It is known that 2%, 3% and 2% of these cars produced from are defective.

   1) What is the probability that a car purchased is defective ?

   2) If a car purchased is found to be defective, what is the probability that this car is produced by company B_1 ? (8)

   ii) Let X be a Poisson variate such that P(X = 1) = 2P(X = 2). Calculate :

   1) $P(X = 0)$ and $P(X > 0.5)$ 2) $P\left(3/2 < X \leq \frac{7}{2}\right)$

   3) $E\left(\frac{3}{2}X + 1\right)$ 4) $Var\left(\frac{1}{2}X - 1\right)$. (8)

(OR)

b) i) The C.D.F. of the R.V. X is given by

$$F(x) = \begin{cases} 0 & , \quad x < -1 \\ \frac{x+1}{2} & , \quad -1 \leq x < 1 \\ 1 & , \quad x \geq 1 \end{cases}$$

   Compute :

   1) $P\left(|X| < 1/4\right)$

   2) $P\left(X > -1/2\right)$ and $P\left(X < \frac{3}{4}\right)$

   3) E(X)

   4) Var (X) . (8)

   ii) Suppose the R.V. X has a geometric distribution

$$P(X = x) = \begin{cases} \left(\frac{1}{2}\right)^x & , \quad x = 1, 2, 3, \ldots \\ 0 & , \quad \text{otherwise} \end{cases}$$

   Obtain :

   1) $P(X \leq 2)$ 2) $P(X > 4/X > 2)$ 3) C.D.F. F(x), of R.V. X. (8)

12. a) i) Let the joint p.m.f. of R.V. (X, Y) be given as

$$P(X=x,Y=y)=\begin{cases}\frac{x+y}{12}, & x=1,2,\ y=1,2\\ 0, & \text{otherwise}\end{cases}$$

Determine :

1) The marginal p.m.f. s of X and Y
2) The conditional p.m.f. $P(X = x/Y = z)$
3) Are the R.V.s X and Y independent ? Justify your answer. (8)

ii) Let X and Y be two independent identically distributed exponential R.V.s with parameter 1. Find the joint p.d.f. of R.V.s $U = X + Y$ and $V = \frac{X}{Y}$ and hence obtain the marginal p.d.f. of R.V. U. (8)

(OR)

b) i) The joint p.d.f. of R.V.s X and Y is given as

$$f(x,y)=\begin{cases}\frac{5y}{4}, & -1\le x\le 1,\ x^2\le y\le 1\\ 0, & \text{otherwise}\end{cases}$$

Find :

1) The marginal p.d.f.s of X and Y
2) Are the R.V.s X and Y independent ? Justify your result. (8)

ii) Suppose the joint p.d.f. of the random variables X and Y is given as

$$f(x,y)=\begin{cases}\frac{1}{49}e^{\frac{-y}{7}}, & 0\le x\le y<\alpha'\\ 0, & \text{otherwise}\end{cases}$$

Compute Cov (X, Y). (8)

13. a) i) A random process is given by $X(t) = U + V\cos(\omega t + \phi)$, where U is a random variable that is uniformly distributed between – 2 and 2, V is a random variable with $E(V) = 0$ and $Var(V) = 2$, ω is a constant and ϕ is a random variable that is uniformly distributed from $-\pi$ to π. Here U, V and ϕ are independent random variables. Is the process X(t) stationary in wide-sense ? Explain. (8)

ii) State and prove the additive property of the Poisson process. (8)

(OR)

b) i) Show that the inter-arrival time between two consecutive arrivals is an exponential random variable. (8)

ii) Discuss the random telegraph signal process X(t) and hence obtain E(X(t)) and $E(X(t)\,X(t+\tau))$. Is the process X(t) a wide-sense stationary ? Explain. (8)

14. a) i) For the jointly wide-sense stationary processes X(t) and Y(t), show that

1) $R_{XY}(-\tau) = R_{YX}(\tau)$

2) $|R_{XY}(\tau)| \leq \frac{1}{2}[R_{XX}(0) + R_{YY}(0)]$

3) $|R_{XY}(\tau)| \leq \sqrt{R_{XX}(0)R_{YY}(0)}$. (8)

ii) A stationary random process X(t) has the power spectral density function $S_{XX}(\omega) = \frac{1}{(4+\omega^2)^2}$. Obtain the correlation function $R_{XX}(\tau)$ and the power of the process X(t). (8)

(OR)

b) i) Find the power spectral density of a wide-sense stationary process with an autocorrelation function $R_{XX}(\tau) = e^{\frac{-\tau^2}{2}}$. (8)

ii) Find the correlation function $R_{XX}(\tau)$ and the average power for spectral density $S_{XX}(\omega) = \frac{3\omega^2 + 4}{2\omega^4 + 6\omega^2 + 4}$. (8)

15. a) i) A random process X(t) is the input to a linear system whose impulse is $h(t) = 2e^{-t}, t \geq 0$. If the auto correlation function of the process X(t) is $R_{XX}(\tau)=e^{-2|\tau|}$, determine the cross-correlation function $R_{XY}(\tau)$. (8)

ii) Let X(t) be the input process to a linear system with autocorrelation function $R_{XX}(\tau) = 3\,\delta(\tau)$ and the impulse response function $h(t) = e^{-bt}, t > 0$. Determine the autocorrelation function of the output process Y(t) and hence obtain $E(Y^2(t))$. (8)

(OR)

b) i) A random process X(t) is applied to a network with impulse response function $h(t) = e^{-bt}, t > 0$, where $b > 0$ is a constant. The cross-correlation of X(t) with output Y(t) is known to have the form $R_{XY}(\tau) = \tau\, e^{-b\tau}, \tau > 0$. Find the auto correlation function of Y(t). (8)

ii) Find the input autocorrelation function, output autocorrelation function and output spectral density of the RC-low pass filter with transfer function $H(\omega) = \frac{1}{1 + j\omega RC}$ and is subject to a white noise of spectral density function $S_{NN}(\omega) = \frac{N_0}{2}$. (8)
