26/11/18 (AN)

Reg. No. : [ ]

Question Paper Code : 20751

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2018.

Third Semester

Civil Engineering

MA 6351 — TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

(Common to All Branches)

(Regulations 2013)

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. Form the partial differential equation by eliminating the arbitrary constants a and b from $z = (x-a)^2 + (y-b)^2 + 1$.

2. Find the complete integral of $p + q = x + y$.

3. Write the complex form of Fourier series in the interval $0 < x < 2\pi$.

4. Find the Root mean square value of the function $f(x) = x - x^2$ in $-1 < x < 1$.

5. Solve $yu_x + xu_y = 0$ using separation of variables method.

6. What are the possible solutions of the one dimensional heat flow equation?

7. State Fourier integral theorem.

8. Prove that $F[f(ax)] = \frac{1}{a} F\left(\frac{s}{a}\right)$, $a > 0$.

9. Find $Z\left[\cos\left(\frac{n\pi}{2}\right)\right]$.

10. State initial and final value theorem for Z-transforms.

PART B — ($5 \times 16 = 80$ marks)

11. (a) (i) Form the partial differential equation by eliminating the arbitrary functions f and g from $z = f(ax+by) + g(\alpha x + \beta y)$. (8)

(ii) Find the general solution of $x(z^2 - y^2)p + y(x^2 - z^2)q = z(y^2 - x^2)$. (8)

Or

(b) (i) Solve $z^2(p^2 + q^2) = x^2 + y^2$. (8)

(ii) Find the general solution of $(D^2 - 6DD' + 5D'^2)z = e^x \sinh y + xy$. (8)

12. (a) (i) Find the Fourier series expansion of $f(x) = x + x^2$ in $-\pi < x < \pi$ and hence deduce the value of $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ldots\ldots\infty$. (8)

(ii) Find the Fourier series expansion of $f(x) = 2x - x^2$ in $0 < x < 3$. (8)

Or

(b) (i) Find the Fourier cosine series expansion of $f(x) = x \sin x$ in $0 < x < \pi$ and hence deduce the value of $1 + \frac{2}{1.3} - \frac{2}{3.5} + \frac{2}{5.7} + \ldots\infty$. (8)

(ii) Compute the first two harmonics of the Fourier series of f(x) from the table : (8)

x	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°	360°
$f(x)$	2.34	3.01	3.68	4.15	3.69	2.20	0.83	0.51	0.88	1.09	1.19	1.64

13. (a) A string is stretched tightly between $x = 0$ and $x = 20$ is fastened at both ends. The midpoint of the string is taken to a height and then released from rest in that position. Find the displacement of any point x of the string at any time t. (16)

Or

(b) A square plate is bounded by the lines $x = 0$, $y = 0$, $x = 20$ and $y = 20$. Its faces are insulated. The temperature along the upper horizontal edge is given by $u(x, 20) = x(20 - x)$ when $0 < x < 20$ while the other three edges are kept at 0°C. Find the steady state temperature in the plate. (16)

14. (a) (i) Find the Fourier Transform of $f(x)$ if $f(x) = \begin{cases} 1 - |x|, & |x| < 1 \\ 0, & |x| > 1 \end{cases}$ and hence evaluate the integral $\int_0^\infty \left(\frac{\sin t}{t}\right)^4 dt$. (10)

(ii) State and prove convolution theorem for Fourier transforms. (6)

Or

(b) (i) Evaluate $\int_0^\infty \frac{dx}{(x^2 + a^2)(x^2 + b^2)}$ using transforms. (6)

(ii) Find the Fourier cosine transform of $f(x) = e^{-a^2x^2}$ and hence find $F_S[xe^{-a^2x^2}]$. (10)

15. (a) (i) Find $Z\left[\frac{1}{(n+1)(n+2)}\right]$. (8)

(ii) Using convolution theorem evaluate $Z^{-1}\left[\frac{8z^2}{(2z-1)(4z+1)}\right]$. (8)

Or

(b) (i) Using Z – Transform solve $y(n+3) - 3y(n+1) + 2y(n) = 0$, with $y(0) = 4$, $y(1) = 0$, $y(2) = 8$. (8)

(ii) Find $Z^{-1}\left[\frac{z}{(z-1)(z^2+1)}\right]$ by using integral method. (8)