Reg. No. : [ ]

16/05/17 AN

Question Paper Code : 72068

B.E./B.Tech. DEGREE EXAMINATION, APRIL/MAY 2017.

Third Semester

Mechanical Engineering

MA 6351 — TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

(Common to Mechanical Engineering (Sandwich)/Aeronautical Engineering/ Agriculture Engineering/Automobile Engineering/B.E. Biomedical Engineering/ B.E. Civil engineering/B.E. Computer Science and Engineering/Electrical and Electronics Engineering/Electronics and Communication Engineering/Electronics and Instrumentation Engineering/Geoinformatics Engineering/Industrial Engineering/Industrial Engineering and Management/Instrumentation and Control Engineering/Manufacturing Engineering/Marine Engineering/Materials Science and Engineering/Mechanical and Automation Engineering/Mechatronics Engineering/Medical Electronics Engineering/Petrochemical Engineering/ Production Engineering/Robotics and Automation Engineering/ Biotechnology/ Chemical Engineering/Chemical and Electrochemical Engineering/Food Technology/Information Technology/Petrochemical Technology/Petroleum Engineering/Plastic Technology/Polymer Technology)

(Regulations 2013)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Form the partial differential equation by eliminating arbitrary function 'f' from $z = e^{ay} f(x+by)$.

2. Solve $(D^3 - D^2D' - 8DD'^2 + 12D'^3)z = 0$.

3. State the sufficient condition for a function $f(x)$ to be expressed as a Fourier series.

4. If the Fourier series of the function $f(x) = x + x^2$, in the interval $(-\pi, \pi)$ is $\frac{\pi^2}{3} + \sum_{n=1}^{\infty}(-1)^n\left[\frac{4}{n^2}\cos nx - \frac{2}{n}\sin nx\right]$, then find the value of the infinite series $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ldots$

5. Write all possible solutions of one dimensional heat equation $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$.

6. Using the method of separation of variables, solve $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$ where $u(x, 0) = 6e^{-3x}$.

7. If $F(s)$ is the Fourier transform of $f(x)$, prove that $F\{f(x-a)\} = e^{ias}F(s)$.

8. Find Fourier Sine transform of $\frac{1}{x}$.

9. Find the Z-transform of a^n.

10. State initial and final value theorems on Z-transforms.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the general solution of $(z^2 - 2yz - y^2)p + (xy + zx)q = xy - zx$. (8)

(ii) Find the general solution of $(D^2 + 2DD' + D'^2)z = x^2 y + e^{x-y}$. (8)

Or

(b) (i) Find the general solution of $z = px + qy + p^2 + pq + q^2$. (8)

(ii) Find the general solution of $(D^2 - 3DD' + 2D'^2 + 2D - 2D')z = \sin(2x + y)$. (8)

12. (a) (i) Find the Fourier series of period 2π for the function $f(x) = x\cos x$ in $0 < x < 2\pi$. (8)

(ii) Find the Fourier series expansion for $y = f(x)$ up to second harmonic from the following data: (8)

x:	0	1	2	3	4	5
y:	9	18	24	28	26	20

Or

(b) (i) Find the Fourier half-range cosine series of $f(x) = \begin{cases} x, & \text{in } 0 < x < 1 \\ 2 - x, & \text{in } 1 < x < 2 \end{cases}$. (8)

(ii) Find the complex form of the Fourier series of $f(x) = e^{-ax}$ in, $-l < x < l$. (8)

13. (a) A tightly stretched string of length $2l$ is fastened at $x = 0$ and $x = 2l$. The midpoint of the string is then taken to height 'b' transversely and then released from rest in that position. Find the lateral displacement of the string. (16)

Or

(b) A rectangular plate with insulated surfaces is 20 cm wide and so long compared to its width that it may be considered infinite in length without introducing an appreciable error. If the temperature while the other short edge $x = 0$ is given by $u = \begin{cases} 10y & \text{for } 0 \le y \le 10 \\ 10(20 - y) & \text{for } 10 \le y \le 20 \end{cases}$ and the two long edges as well as the other short edge are kept at 0°c, find the steady state temperature distribution $u(x, y)$ in the plate. (16)

14. (a) Find the Fourier transform of $f(x)$ given by $f(x) = \begin{cases} 1 & \text{for } |x| < 2 \\ 0 & \text{for } |x| > 2 \end{cases}$ and hence evaluate $\int_0^\infty \frac{\sin x}{x} dx$ and $\int_0^\infty \left(\frac{\sin x}{x}\right)^2 dx$. (16)

Or

(b) (i) Find the Fourier cosine transform of $e^{-a^2x^2}$ for any $a > 0$. (8)

(ii) Evaluate $\int_0^\infty \frac{dx}{(x^2+1)(x^2+4)}$ using Fourier transforms. (8)

15. (a) (i) Find Z-transform of $\frac{2n+3}{(n+1)(n+2)}$. (8)

(ii) Using Convolution theorem, find $Z^{-1}\left[\frac{8z^2}{(2z-1)(4z+1)}\right]$. (8)

Or

(b) (i) Find $Z^{-1}\left[\frac{4z^3}{(2z-1)^2(z-1)}\right]$, by the method of partial fractions. (8)

(ii) Using Z-transforms, solve the equation $y_{n+2} - 7y_{n+1} + 12y_n = 2^n$, given that $y_0 = y_1 = 0$. (8)