Reg. No. :

Question Paper Code : 90337

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2019

Third Semester

Medical Electronics

MA8352 – LINEAR ALGEBRA AND PARTIAL DIFFERENTIAL EQUATIONS

(Common to : Biomedical Engineering/Computer and Communication Engineering/ Electronics and Communication Engineering/Electronics and Telecommunication Engineering)

(Regulations 2017)

Time : Three Hours — Maximum : 100 Marks

Answer ALL questions

PART – A (10×2=20 Marks)

1. If $V = R^3$, then verify whether $W = \{(a_1, a_2, a_3)/2a_1 - 7a_2 + a_3 = 0\}$ is a subspace or not.

2. Find the dimension of W, where $W = \{(x_1, x_2, x_3)/x_1 + x_2 + x_3 = 0\}$.

3. Let $T : P_3(R) \to P_2(R)$ be a linear transformation defined by $T(f(x)) = f'(x)$. Let B_1 and B_2 be the standard bases for $P_3(R)$ and $P_2(R)$ respectively. Then find [T].

4. Test the matrix $A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \in M_{2\times 2}(R)$ for diagonalizable.

5. Let $V = R^2$ and $S = \{(1,0), (0, 1)\}$. Check whether S is orthonormal basis or not.

6. Find the conjugate transpose of $A = \begin{pmatrix} i & 1+2i \\ 2 & 3+4i \end{pmatrix}$.

7. Form the partial differential equation by eliminating the arbitrary function from $z = e^{x-y} \cdot f(x+y)$.

8. Find the complete integral of the partial differential equation $z = px + qy + p^2 - q^2$.

9. State Dirichlet's conditions for Fourier series of f(x) defined in the interval $c \le x \le c + 2l$.

10. Write all three possible solutions of one dimensional heat equation.

PART – B **(5×16=80 Marks)**

11. a) i) Determine the given set in $P_4(R)$ is linearly dependent or linearly independent for $x^4 - x^3 + 5x^2 - 8x + 6$, $-x^4 + x^3 - 5x^2 + 5x - 3$, $x^4 + 3x^2 - 3x + 5$ and $2x^4 + x^3 + 4x^2 + 8x$. **(8)**

ii) Let $S = \{v_1, v_2, v_3\}$ where $v_1 = (1, -3, -2)$, $v_2 = (-3, 1, 3)$, $v_3 = (-2, -10, -2)$. Verify whether S forms a basis or not. **(8)**

(OR)

b) i) Verify whether the first polynomial can be expressed as a linear combination of the other two in P_3 (R) for the given $x^3 - 8x^2 + 4x$, $x^3 - 2x^2 + 3x - 1$ and $x^3 - 2x + 3$. **(8)**

ii) Let W_1 and W_2 be subspaces of V. Prove that $W_1 \cup W_2$ is a subspace of V if and only if $W_1 \subseteq W_2$ (or) $W_2 \subseteq W_1$. **(8)**

12. a) i) Let $T : R^3 \to R^2$ be defined by $T(x, y, z) = (2x, -y, 3z)$. Verify whether T is linear or not. Find N(T) and R(T) and hence verify the dimension theorem. **(8)**

ii) Let $T : P_2(R) \to P_2(R)$ be defined as $T[f(x)] = f(x) + (x + 1) f'(x)$. Find eigenvalues and corresponding eigenvectors of T with respect to standard basis of $P_2(R)$. **(8)**

(OR)

b) i) Test for diagonalizability of the matrix $A = \begin{bmatrix} 7 & -4 & 0 \\ 8 & -5 & 0 \\ 6 & -6 & 0 \end{bmatrix}$ and if A is diagonalizable, find the invertible matrix Q such that $Q^{-1}AQ = D$. **(8)**

ii) Let T be the linear operator on R^3 defined by $T\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{bmatrix} 4a_1 + a_3 \\ 2a_1 + 3a_2 + 2a_3 \\ a_1 + 4a_3 \end{bmatrix}$.

Determine the eigenspace of T corresponding to each eigenvalue. Let B be the standard ordered basis for R^3. **(8)**

13. a) i) Let R^3 have the Euclidean inner product. Use Gram-Schmidth process to transform the basis $\{u_1, u_2, u_3\}$ into an orthonormal basis, where $u_1 = (1, 1, 1)$, $u_2 = (0, 1, 1)$ and $u_3 = (0, 0, 1)$. **(10)**

ii) Let $S = \{(1, 1, 0), (1, -1, 1), (-1, 1, 2)\}$ be an orthogonal set then orthonormal set is $\left\{\frac{1}{\sqrt{2}}(1,1,0), \frac{1}{\sqrt{3}}(1,-1,1), \frac{1}{\sqrt{6}}(-1,1,2)\right\}$ both are basis of R^3. Let $x = (2, 1, 3) \in R^3$. Express x as a linear combination of orthogonal set S and orthonormal set. **(6)**

(OR)

b) i) Use the least square approximation to find the best fit with a linear function and hence compute the error for the following data (–3, 9), (–2, 6), (0, 2) and (1, 1). **(10)**

ii) Compute the orthogonal complement of $S = \{(1, 0, i), (1, 2, 1)\}$ in C^3. **(6)**

14. a) i) Solve $z = p^2 + q^2$. **(8)**

ii) Find the complete integral of $p^2y(1 + x^2) = qx^2$. **(8)**

(OR)

b) i) Solve $p\sqrt{x} + q\sqrt{y} = \sqrt{z}$. **(8)**

ii) Solve $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 6\frac{\partial^2 z}{\partial y^2} = x + y$. **(8)**

15. a) i) Find the cosine series for $f(x) = x - x^2$ in the interval $0 < x < 1$. **(8)**

ii) Obtain the sine series for $f(x) = x$ in $0 < x < \pi$ and hence deduce that $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$. **(8)**

(OR)

b) i) An finitely long uniform plate is bounded by two parallel edges and an end at right angles to them. The breadth is π. This end is maintained at a temperature u_0 at all points and other edges are kept at zero temperature. Determine the temperature at any point of the plate in the steady state. **(8)**

ii) A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially in a position given by $y(x, 0) = y_0 \sin^3\left(\frac{\pi x}{l}\right)$. If it is released from rest from this position, find the displacement y at any time and at any distance from the end $x = 0$. **(8)**
