Reg. No. : [ ][ ][ ][ ][ ][ ][ ][ ][ ][ ][ ][ ]

Question Paper Code : 20758

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2018.

Fifth Semester

Computer Science and Engineering

MA 6566 — DISCRETE MATHEMATICS

(Regulations 2013)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Write the contra positive of the implication. "If it is Sunday then it is holiday".

2. Show that the propositions $p \to q$ and $\neg p \vee q$ are equivalent.

3. How many cards must be selected from a deck of 52 cards to guarantee that at least three cards of the same suit are chosen?

4. How many bit strings of length 12 contain exactly four 1s?

5. Show that the number of odd degree vertices in a simple graph is even.

6. Give an example of a graph which is both Eulerian and Hamiltonian.

7. Define a semigroup and give an example.

8. Show that in a group $(G,*)$ if for any $a,b \in G$, $(a*b)^2 = a^2 * b^2$, then $(G,*)$ is abelian.

9. Draw the Hasse diagram of $(S_{24},/)$ where S_{24} denotes the set of positive divisors of 24 and / denotes the relation "division".

10. Prove that in a lattice $(L,\leq)$, $a*(a \oplus b) = a$ where $*$ and $\oplus$ denote the meet and join.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Translate the statement $\forall x\,(C(x) \vee \exists y\,(C(y) \wedge F(x,y)))$ into English, where $C(x)$ is "x has a computer", $F(x,y)$ is "x and y are friends" and the universe of discourse for both x and y consists of all students in your class. (4)

(ii) Translate the statement "The sum of two positive integers is a positive integer" into a logical expression. (4)

(iii) Show that the premises, "A student in this class has not read the book" and "Everyone in this class passed the exam" imply the conclusion "Someone who passed the exam has not read the book". (8)

Or

(b) (i) Obtain the principal disjunctive and conjunctive normal forms of the formula $(\sim p \rightarrow r) \wedge (q \leftrightarrow p)$. (8)

(ii) Using proof by contradiction, prove that $\sqrt{2}$ is irrational. (8)

12. (a) (i) Use mathematical induction to show that $n^2 - 1$ is divisible by 8 whenever n is an odd positive integer. (8)

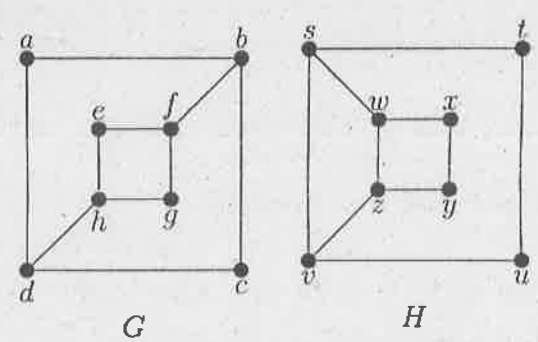
(ii) Solve the recurrence relation $f_n = f_{n-1} + f_{n-2}$ with $f_0 = 0$; $f_1 = 1$. (8)

Or

(b) (i) Using generating functions, solve $a_n = 8a_{n-1} + 10^{n-1}$ with $a_0 = 1; a_1 = 9$. (8)

(ii) How many onto functions are there from a set with six elements to set with three elements? (8)

13. (a) (i) Determine whether the graphs given below are isomorphic. (8)



(ii) Let G be a simple graph with adjacency matrix A. Show that the number of different walks of length r from v_i to v_j, where r is a positive integer, equals the $(i,j)^{th}$ entry of A^r. (8)

Or

(b) (i) Show that a connected simple graph is Eulerian if and only if all its vertices have even degree. (8)

(ii) Represent each of the following graphs with an adjacency matrix.

(1) K_4

(2) $K_{1,4}$

(3) C_4

(4) W_4. (8)

14. (a) (i) State and prove Lagrange's theorem on groups. (12)

(ii) Show that if every element in a group is its own inverse, then the group must be abelian. (4)

Or

(b) (i) Show that a subset $S \neq \phi$ of G is a subgroup of the group $(G,*)$ if and only if for any pair of elements $a, b \in S, a * b^{-1} \in S$. (8)

(ii) Let f be a group homomorphism from $(G,*)$ to (H,Δ). Define Kernel of f and show that it is a subgroup of $(G,*)$. (8)

15. (a) (i) Show that every chain is a distributive lattice. (8)

(ii) Show that every distributive lattice is modular, but not conversely. (8)

Or

(b) (i) Show that the following are equivalent in a Boolean Algebra $a \leq b \Leftrightarrow a * b' = 0 \Leftrightarrow b' \leq a' \Leftrightarrow a' \oplus b = 1$. (8)

(ii) In a Boolean algebra, prove that $(a * b)' = a' \oplus b'$ and $(a \oplus b)' = a' * b'$. (8)