





## PART - B

(5×16=80 Marks)

11. a) i) Prove that the number of vertices of odd degree in a graph is always even. (6)
- ii) Prove that a connected graph  $G$  is an Euler graph if and only if it can be decomposed into circuits. (10)
- (OR)
- b) i) Prove that a tree with  $n$  vertices has  $(n - 1)$  edges. (6)
- ii) State and prove Dirac's theorem. (10)
12. a) i) Prove that every circuit has an even number of edges in common with a cut-set. (8)
- ii) Prove the following :  
With respect to the given spanning tree  $T$ , a branch  $b_i$  that determines a fundamental cut-set  $S$  is contained in every fundamental circuit associated with the chords in  $S$  and in no others. (8)
- (OR)
- b) i) Explain max-flow min-cut theorem. (8)
- ii) Define 2-isomorphism and prove that the rank and nullity of a graph are invariant under 2-isomorphism. (8)
13. a) i) If  $G$  is a tree with  $n$  vertices, then prove that its chromatic polynomial is  $P_n(\lambda) = \lambda(\lambda - 1)^{n-1}$ . (8)
- ii) Define chromatic number. Prove that a graph with at least one edge is 2-chromatic if and only if it has no circuits of odd length. (8)
- (OR)
- b) i) State and prove five-colour theorem. (8)
- ii) Discuss about any four types of digraph with suitable examples. (8)
14. a) i) Using the principle of inclusion and exclusion find the number of prime numbers not exceeding 100. (8)
- ii) Show that if  $n$  and  $k$  are positive integers, then  $C(n+1, k) = \frac{n+1}{k} C(n, k-1)$ . Use this identity, construct an inductive definition of the binomial co-efficient. (8)
- (OR)



- b) i) A survey of 150 college students reveals that 83 own cars, 97 own bikes, 28 own motorcycles, 53 own a car and a bike, 14 own a car and motorcycle, 7 own a bike and a motorcycle and 2 own all the three. How many students own a bike and nothing else and how many students do not own any of the three? (8)
- ii) Five professors  $P_1, P_2, P_3, P_4, P_5$  are to be made class advisor for five sections  $C_1, C_2, C_3, C_4, C_5$ , one professor for each section.  $P_1$  and  $P_2$  do not wish to become the class advisors for  $C_1$  or  $C_2$ ,  $P_3$  and  $P_4$  for  $C_4$  or  $C_5$  and  $P_5$  for  $C_3$  or  $C_4$  or  $C_5$ . In how many ways can the professors be assigned the work (without displacing any professor)? (8)
15. a) i) Obtain the fractional de-composition and identify the sequence having the expression  $\frac{3-5z}{1-2z-3z^2}$  as a generating function. (8)
- ii) Find the generating function of the sequence 7, 8, 9, 10, ... (4)
- iii) Find the number of distinct summands of the integer 6. (4)
- (OR)
- b) i) Solve the recurrence relation  $y_{n+2} - 6y_{n+1} + 8y_n = 3n+5$ . (8)
- ii) If  $a_n$  denotes the sum of the first  $n$  positive integers, find a recurrence relation for  $a_n$  and then solve it. (8)