(b) (i) The autocorrelation function of the random telegraph signal process is given by $R(\tau) = \alpha^2 e^{-2\gamma|\tau|}$. Determine the power spectral density of the random telegraph signal. (8)

(ii) Find the mean square value of the processes whose power spectral density is given by $S_X(\omega) = 1/(\omega^4 + 10\omega^2 + 9)$. (8)

15. (a) (i) Let $X(t)$ be the input voltage to a circuit and $Y(t)$ be the output voltage. $\{X(t)\}$ is a stationary random process with $\mu_X = 0$ and $R_{XX}(\tau) = e^{-\alpha|\tau|}$. Find μ_Y, $S_{YY}(\omega)$ if the power transfer function is $H(\omega) = \dfrac{R}{R + iL\omega}$. (8)

(ii) If a system is such that its input $X(t)$ and its output $Y(t)$ are related by a convolution integral, prove that the system is a linear time invariant system. (8)

Or

(b) A random process $X(t)$ is the input to a linear system whose impulse response is given by $h(t) = 2e^{-t}$, $t \geq 0$. If the autocorrelation function of the process $X(t)$ is $R_{XX}(\tau) = e^{-2|\tau|}$, determine the cross correlation function $R_{XY}(\tau)$ and $R_{YX}(\tau)$ between the input process $X(t)$ and the output process $Y(t)$. (16)

———————

30/11/18

FN

Reg. No. :

Question Paper Code : 20752

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2018.

Fourth Semester

Electronics and Communication Engineering

MA 6451 — PROBABILITY AND RANDOM PROCESSES

(Common to Biomedical Engineering, Robotics and Automation Engineering)

(Regulations 2013)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Prove that $P[X = x] = \frac{2}{3}(1/3)^x$: $x = 0, 1, 2, \ldots$ is a probability mass function of a discrete random variable X.

2. Find the mean of the random variable X whose moment generating function is given by $\frac{1}{81}(2t+1)^4$.

3. The equations of two regression lines of random variables X, Y are $4x - 5y + 33 = 0$, $20x - 9y - 107 = 0$. Find the mean values of X and Y.

4. Find the value of k if $f(x, y) = \begin{cases} k\,x\,y\,e^{-(x^2+y^2)}; & x \geq 0, y \geq 0 \\ 0; & \text{otherwise} \end{cases}$ is a joint probability density function of (X, Y).

5. Give an example of an evolutionary process.

6. State any two properties of a Poisson process.

7. Prove that the auto correlation function is an even function of τ.

8. Compute the mean value of the random process $\{X(t)\}$ whose autocorrelation function is given by $R(\tau) = 25 + 4/(1+6\tau^2)$.

9. Check whether the function $y(t) = x^2(t)$ is linear.

10. When is a system said to be stable?

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the mean and variance of the random variable X if the probability density function of X is given by

$$f(x) = \begin{cases} x & : 0 < x < 1 \\ 2 - x & : 1 < x < 2 \\ 0 & : \text{otherwise} \end{cases}$$ (8)

(ii) Find the moment generating function of the Poisson distribution and hence find its mean and variance. (8)

Or

(b) (i) The mileage which car owners get with a certain kind of radial tire is a random variable having an exponential distribution with mean 40,000 km. Find the probability that one of these tires will last at least 20,000 km. Also find the probability that one of these tires will last at most 30,000 km. (8)

(ii) State and prove the memory less property of Geometric distribution. (8)

12. (a) (i) Find the marginal density functions of X and Y if the joint probability density function is

$$f(x, y) = \begin{cases} \frac{2}{5}(2x + 3y); & 0 \le x \le 1,\ 0 \le y \le 1 \\ 0 & ; \text{otherwise.} \end{cases}$$ (8)

(ii) Calculate the coefficient of correlation between the variables X and Y from the data given below. (8)

X	64	65	66	67	68	69	70
Y	66	67	65	68	70	68	72

Or

(b) (i) If X and Y each follow an exponential distribution with parameter 1 and are independent, find the probability density function of $U = X - Y$. (8)

(ii) Find $E(XY)$ if the joint probability density function of two dimensional random variables (X, Y) is given by $f(x, y) = 24y(1-x)$: $0 \le y \le x \le 1$. (8)

13. (a) (i) Calculate the autocorrelation function of the process $X(t) = A\sin(\omega t + \theta)$, where A and ω are constants and θ is uniformly distributed random variable in $(0, 2\pi)$. (8)

(ii) A radioactive source emits particles at a rate of 6 per minute in accordance with Poisson process. Each particle emitted has a probability of 1/3 of being recorded. Find the probability that 5 particles are recorded in 5 minute period. (8)

Or

(b) (i) Two random processes $X(t)$ and $Y(t)$ are defined by $X(t) = A\cos t + B\sin t$ and $Y(t) = B\cos t - A\sin t$. Find the cross correlation function of $X(t)$ and $Y(t)$ if A and B are uncorrelated random variables with zero means and the same variance. (8)

(ii) A man either drives a car or catches a train to go to office each day. He never goes two days in a row by train but if he drives one day, then the next day he is just as likely to drive again as he is to travel by train. Now suppose that on the first day of the week, the man tossed a fair die and drove to work if and only if 6 appeared. Find the probability transition matrix. Obtain the probability that he takes a train on the third day. (8)

14. (a) (i) Using the autocorrelation function

$$R_{XX}(\tau) = 75e^{-10|\tau|} + 25\cos(20\tau) + 49,$$

find the mean, mean square value and the variance of the random process. (8)

(ii) Two random processes $\{X(t)\}$ and $\{Y(t)\}$ are defined as $X(t) = A\cos(\omega t + \theta)$ and $Y(t) = B\sin(\omega t + \theta)$, where A, B, ω are constants and θ is uniformly distributed random variable in $(0, 2\pi)$. Find the cross correlation function of $\{X(t)\}$ and $\{Y(t)\}$. (8)

Or