

21/05/18

P21



Reg. No. :

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Question Paper Code : 41303

B.E./B.Tech. DEGREE EXAMINATION, APRIL/MAY 2018

First Semester

Mechanical Engineering

MA 6151 – MATHEMATICS – I

Common to Mechanical Engineering (Sandwich) Aeronautical Engineering/
Agriculture Engineering/Automobile Engineering/Biomedical Engineering/Civil
Engineering/Civil Engineering and Computer Based Construction/Computer
Science and Engineering/Computer and Communication Engineering/Electrical
and Electronics Engineering/Electronics and Communication Engineering/
Electronics and Instrumentation Engineering/Environmental Engineering/
Geoinformatics Engineering/Industrial Engineering/Industrial Engineering
and Management/Instrumentation and Control Engineering/Manufacturing
Engineering/Material Science and Engineering/Mechanical and Automation
Engineering/Mechatronics Engineering/Medical Electronics/Metallurgical
Engineering/Petrochemical Engineering/Production Engineering/Robotics and
Automation Engineering/B.E./B.Tech. (Common to all Branches except Marine
Engg.)/Bio Technology/Chemical Engineering/Chemical and Electrochemical
Engineering/Fashion Technology/Food Technology/Handloom and Textile
Technology/Industrial Bio Technology/Information Technology/Leather
Technology/ Petrochemical Technology/ Petroleum Engineering/Pharmaceutical
Technology/Plastic Technology/Polymer Technology/Rubber and Plastics
Technology/Textile Chemistry/Textile Technology/Textile Technology (Fashion
Technology)/Textile Technology (Textile Chemistry)

(Regulations 2013)

Time : Three Hours

Maximum : 100 Marks

Answer ALL questions.

PART – A

(10×2=20 Marks)

1. Find the eigen values of the matrix $A = \begin{pmatrix} 1 & 3 & 4 \\ 0 & 2 & 5 \\ 0 & 0 & 3 \end{pmatrix}$ and hence find the eigen values of A^{-1} .
2. Discuss the nature of the quadratic form $2x^2 + 2xy + 3y^2$.



3. State the necessary condition for the convergence of series of positive terms.
4. Define absolutely convergent and conditionally convergent of a series.
5. Find the curvature of the curve $2x^2 + 2y^2 + 5x - 2y + 1 = 0$.
6. List two important properties of the evolute.
7. If $x = r^2 - \theta^2$, $y = 2r\theta$ find $\frac{\partial r}{\partial x}$.
8. When is a function said to be stationary at a point (x, y) ?
9. Evaluate $\int_{-1}^2 \int_x^{x+2} dy dx$.
10. Evaluate $\int_0^{2\pi} \int_0^{\frac{\pi}{2}} \int_0^4 r^3 \sin \theta dr d\theta d\phi$.

PART - B

(5×16=80 Marks)

11. a) Verify Cayley-Hamilton theorem for the matrix $A = \begin{pmatrix} 1 & 2 & -2 \\ 2 & 5 & -4 \\ 3 & 7 & -5 \end{pmatrix}$ and hence find A^{-1} . (10 + 6)
- (OR)
- b) Reduce the following quadratic form to a canonical form by orthogonal reduction and find the rank, index signature and the nature of the quadratic form : $(-x^2 + y^2 + 4yz + 4zx)$. (8+2+2+2+2)
12. a) i) Use integral test to check the convergence of the series $\sum_{n=1}^{\infty} \frac{1}{x^n + x^{-n}}$ (8)
- ii) Test for the convergence of the series $\sum_{n=1}^{\infty} \frac{n^2}{3^n}$ by D'Alembert's Ratio test. (8)
- (OR)
- b) i) Discuss the convergence of the series $\frac{5}{2} - \frac{7}{4} + \frac{9}{6} - \frac{11}{8} + \dots$ by Leibnitz's rule. (8)
- ii) Test $\sum_{n=2}^{\infty} \frac{(-1)^n}{n(\log n)^2}$ for convergence and absolute convergence. (8)



13. a) i) Find the circle of the curvature at $(0, 0)$ on $x + y = x^2 + y^2 + x^3$. (8)
- ii) Find the evolute of the four cusped hypocycloid $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ (8)
- (OR)
- b) i) Find the envelope of $\frac{x}{a} + \frac{y}{b} = 1$ subject to $a^n + b^n = c^n$ given c is a known constant. (8)
- ii) Considering the evolute of a curve as the envelope of the normals, find the evolute of $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. (8)
14. a) i) If $f_1 = u - x - y - z = 0$, $f_2 = uv - y - z = 0$, $f_3 = uvw - z = 0$ then prove that $\frac{\partial(x, y, z)}{\partial(u, v, w)} = u^2 v$ (8)
- ii) Find the Taylor's series expansion for $f(x, y) = x^2 + y^2 + 2xy$ at $(1, 1)$ upto second degree terms. (8)
- (OR)
- b) i) Find the maxima and minima of $xy(a - x - y)$. (8)
- ii) The temperature $u(x, y, z)$ at any point in space is $u = 400xyz^2$. Find the highest temperature on surface of the sphere $x^2 + y^2 + z^2 = 1$. (8)
15. a) i) Change the order of integration in $I = \int_{-a}^a \int_0^{\sqrt{a^2 - x^2}} (x^2 + y^2) dy dx$. (4)
- ii) Evaluate $\iint_A (x^2 + y^2) dx dy$ where A is the area bounded by the curves $x^2 = y$, $x = 1$, $x = 2$ and the x axis. (12)
- (OR)
- b) i) Evaluate $\int_0^{\infty} \int_0^{\infty} e^{-(x^2 + y^2)} dx dy$ and hence evaluate $\int_0^{\infty} e^{-x^2} dx$. (6+2)
- ii) Find the volume of the tetrahedron bounded by the coordinate planes and $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$. (8)