

22/05/18



Reg. No. :

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Question Paper Code : 41307

22/05/18

(Ans)

**B.E./B.Tech. DEGREE EXAMINATION, APRIL/MAY 2018
Second Semester**

**Mechanical Engineering
MA 6251 – MATHEMATICS – II**

**(Common to Mechanical Engineering (Sandwich)/Aeronautical Engineering/
Agriculture Engineering/Automobile Engineering/Biomedical Engineering/
Civil Engineering/Computer Science and Engineering/Computer and
Communication Engineering/Electrical and Electronics Engineering/Electronics
and Communication Engineering /Electronics and Instrumentation Engineering/
Environmental Engineering/Geoinformatics Engineering/Industrial Engineering/
Industrial Engineering and Management/Instrumentation and Control
Engineering/Manufacturing Engineering/Materials Science and Engineering/
Mechanical and Automation Engineering/Mechatronics Engineering/Medical
Electronics/Petrochemical Engineering/Production Engineering/Robotics and
Automation Engineering/(Common to all Branches except Marine Engg.)/Bio
Technology/Chemical Engineering/Chemical and Electrochemical Engineering/
Fashion Technology/Food Technology/Handloom and Textile Technology/
Information Technology/Petrochemical Technology/Petroleum Engineering/
Pharmaceutical Technology/Plastic Technology/Polymer Technology/Textile
Chemistry/Textile Technology/Textile Technology (Fashion Technology)
(Regulations 2013)**

Time : Three Hours

Maximum : 100 Marks

Answer ALL questions.

PART – A (10×2=20 Marks)

1. In what direction from (-1, 1, 2) is the directional derivative of $\phi = xy^2z^3$ a maximum ?
2. Find the value of 'a' for the vector $\vec{F} = (2x^2y + yz) \vec{i} + (xy^2 - xz^2) \vec{j} + (axyz - 2x^2y^2) \vec{k}$ to be solenoidal.
3. Find the complementary function $\frac{d^3y}{dx^3} - 3\frac{d^2y}{dx^2} + 4\frac{dy}{dx} - 2y = 0$.
4. Write the general form of Cauchy's homogeneous linear equation.

5. If $f(t) = \begin{cases} 3, & 0 < t < 2 \\ -1, & 2 < t < 4 \\ 0, & t \geq 4 \end{cases}$, find $L[f(t)]$.
6. State and prove change of scale property.
7. Verify whether $w = (x^2 - y^2 - 2xy) + ix^2 - y^2 + 2xy$ is an analytic function of $z = x + iy$.
8. Define conformal mapping.
9. Evaluate $\int_C (z^2 - z + 1) dz$ where C is the circle $|z| = 2$.
10. Write the Laurent's series expansion.

PART - B (5×16=80 Marks)

11. a) i) Prove that $\text{div grad } r^n = n(n+1)r^{n-2}$. (8)
- ii) Find the work done in moving a particle in the force field $\vec{F} = 3x^2 \vec{i} + (2xz - y) \vec{j} + z \vec{k}$ along the straight line from $(0, 0, 0)$ to $(2, 1, 3)$. (8)
- (OR)
- b) i) Evaluate $\oint_C [xy + x^2] dx + [x^2 + y^2] dy$ where C is the square formed by the lines $x = 1, x = -1, y = 1, y = -1$ using Green's theorem in the plane. (6)
- ii) Verify Stoke's theorem for $\vec{F} = y^2 \vec{i} + y \vec{j} - xz \vec{k}$ over the upper half of the sphere $x^2 + y^2 + z^2 = a^2, z \geq 0$. (10)
12. a) i) Solve: $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = \log x \sin(\log x)$. (8)
- ii) Solve by the method of variation of parameters $\frac{d^2y}{dx^2} - 3 \frac{dy}{dx} + 2y = \frac{e^x}{1+e^x}$. (8)
- (OR)
- b) i) Solve $(3x+2)^2 \frac{d^2y}{dx^2} + 3(3x+2) \frac{dy}{dx} - 36y = 3x^2 + 4x + 1$. (8)
- ii) Solve the system of equations $\frac{dx}{dt} + 2y = -\sin t; \frac{dy}{dt} - 2x = \cos t$. (8)

13. a) i) Find the Laplace transform of $e^{-t} t^2 \sin 2t$. (8)
- ii) Obtain the Laplace transform of the periodic saw-tooth wave function given by $f(t) = \frac{kt}{\omega}$ for $0 < t < \omega$, and $f(t + \omega) = f(t)$. (8)
- (OR)
- b) i) Using convolution theorem, find $L^{-1} \left\{ \frac{1}{s^3(s+1)} \right\}$. (8)
- ii) Solve $(D^2 - D - 2)y = 20 \sin 2t$ given that $y = -1, Dy = 2$ when $t = 0$ by using Laplace transform methods. (8)
14. a) i) Find the harmonic conjugate of the function $v(x, y) = e^x [x \sin y + y \cos y]$ if $f(z) = u + iv$. (8)
- ii) Construct the analytic function $f(z) = u(r, \theta) + iv(r, \theta)$. Given that $u(r, \theta) = r^2 \cos 2\theta - r \cos \theta + 2$. (8)
- (OR)
- b) i) Find the image of the line $y = 3x + 1$ under the transformation $w = z^2$. (6)
- ii) Find the bilinear transformation which maps the points $z = 1, i, -1$ into the points $w = i, 0, -i$. Hence find the image of $|z| < 1$. (10)
15. a) i) Evaluate $\int_C \frac{z-1}{(z+1)^2(z-2)} dz$ where C is the circle $|z-i|=2$ by Cauchy's Integral Formula. (8)
- ii) Find the Taylor's series expansion for the function $f(z) = \frac{1}{(1+z)^2}$ about $z = -i$. (8)
- (OR)
- b) i) Using Cauchy's residue theorem, evaluate $C \int_C \frac{4-3z}{z(z-1)(z-2)} dz$ where C is the circle $|z| = \frac{3}{2}$. (8)
- ii) Prove that $\int_0^{2\pi} \frac{d\theta}{2 + \cos \theta} = \frac{2\pi}{\sqrt{3}}$ using contour integration. (8)