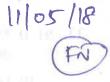


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## Question Paper Code: 41312



## B.E./B.Tech. DEGREE EXAMINATION, APRIL/MAY 2018 Fourth Semester

Electronics and Communication Engineering
MA 6451 – PROBABILITY AND RANDOM PROCESSES

(Common to Biomedical Engineering, Robotics and Automation Engineering)
(Regulations 2013)

Time: Three Hours

Maximum: 100 Marks

Answer ALL questions

THE WEST OF BEING THE SHEET HERE THE PART - A

 $(10\times2=20 \text{ Marks})$ 

- 1. If  $f(x) = \frac{x^2}{3}$ , -1 < x < 2 is the pdf of the random variable X, then find P[0 < x < 1].
- 2. Messages arrive at a switchboard in a Poisson manner at an average rate of six per hour. Find the probability for exactly two messages arrive within one hour.
- 3. If X and Y are random variables having the joint density function  $f(x, y) = \frac{1}{8}(6 x y)$ , 0 < x < 2, 2 < y < 4, then find P[X + Y < 3].
- 4. Find the acute angle between the two lines of regression.
- 5. Define Markov process.
- 6. State any two properties of Poisson process.
- 7. Find the mean square value of the random process  $\{X(t)\}$  if its autocorrelation function  $R(\tau) = 25 + \frac{4}{1+6\tau^2}$ .
- 8. Write any two properties of the power spectral density of the WSS process.
- 9. Prove that the mean of the output process is the convolution of the mean of the input process and the impulse response.
- 10. Assume that the input X(t) to a linear time-invariant system is white noise. What is the power spectral density of the output process Y(t) if the system response  $H(\omega) = 1$ ,  $\omega_1 < |\omega| < \omega_2$  is given?

  = 0, otherwise

**(6)** 

PART - B

 $(5\times16=80 \text{ Marks})$ 

r a uniform random variable X in the interval (a, b), derive the moment nerating function and hence obtain its mean and variance. (10)

t X be the random variable that denotes the outcome of the roll of a fair e. Compute the mean and variance of X. (6)

(OR)

nd the moment generating function of Gamma distribution with parameters and  $\lambda$  and hence compute the first four moments. (10)

continuous random variable X has the density function f(x) given by

 $(x) = \frac{k}{x^2 + 1}$ ,  $-\infty < x < \infty$ . Find the value of k and the cumulative distribution

 $f(x, y) = \frac{1}{8}(x + y), 0 \le x \le 2, 0 \le y \le 2$  is the joint pdf of X and Y. Obtain

correlation coefficient between X and Y.

(OR)

et (X, Y) be a two dimensional non-negative continuous random variable aving the joint probability density function  $f(x, y) = 4xy e^{-(x^2 + y^2)}, x \ge 0,$   $\ge 0$ . Find the probability density function of  $\sqrt{X^2 + Y^2}$ . (10)

ind P[X < Y/X < 2Y] if the joint pdf of (X, Y) is  $f(x, y) = e^{-(x+y)}$ ,  $0 \le x < \infty$ ,  $\le y < \infty$ .

rove that Poisson process is a Markov process. (8)

random process  $\{X(t)\}$  is defined by  $X(t) = A \cos t + B \sin tt$ ,  $-\infty < t < \infty$ , where A and B are independent random variables each of which has a value -2 with probability  $\frac{1}{3}$  and a value 1 with probability  $\frac{2}{3}$ . Show that  $\{X(t)\}$  is

ot stationary in strict sense.

(OR)

 $\{X_1(t)\}\$  and  $\{X_2(t)\}\$  represent two independent Poisson processes with parameters  $\lambda_1 t$  and  $\lambda_2 t$  respectively, then prove that  $P[X_1(t) = x/\{X_1(t) + X_2(t) = n\}]$ 

s binomial with parameters n and p, where  $p = \frac{\lambda_1}{\lambda_1 + \lambda_2}$ . (10)

Consider a random process  $\{X(t)\}$  such that  $X(t) = A \cos(\omega t + \theta)$ , where A and  $\omega$  are constants, and  $\theta$  is a uniform random variable distributed with nterval  $(-\pi, \pi)$ . Check whether the process  $\{X(t)\}$  is a stationary process in wide sense.

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14. a) i) Consider two random processes  $X(t) = 3\cos(\omega t + \theta)$  and  $Y(t) = 2\cos(\omega t + \theta - \frac{\pi}{2})$ ,

where  $\theta$  is a random variable uniformly distributed in  $(0, 2\pi)$ . Prove that

 $\sqrt{R_{XX}(o) R_{YY}(o)} \ge |R_{XY}(\tau)|. \tag{10}$ 

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ii) Determine the autocorrelation function of the random process with the power spectral density given by  $S_{XX}(\omega) = S_0$ ,  $|\omega| < \omega_0$ . (6)

(OR)

b) i) Given that a process  $\{X(t)\}$  has the autocorrelation function  $R_{XX}(\tau) = Ae^{-\alpha - |\tau|} \cos{(\omega_0 \tau)} \text{ where } A > 0, \alpha > 0 \text{ and } \omega_0 \text{ are real constants, find the power spectrum of } X(t). \tag{8}$ 

ii) The cross-power spectrum of real random processes X(t) and Y(t) is given by  $S_{XY}(\omega) = a + jb\omega$ ,  $|\omega| < 1$ . Find the cross-correlation function. (8)

15. a) i) Show that  $S_{YY}(\omega) = |H(\omega)|^2 S_{XX}(\omega)$ , where  $S_{XX}(\omega)$  and  $S_{YY}(\omega)$  are the power spectral density functions of the input X(t) and the output Y(t) and  $H(\omega)$  is the system transfer function. (8)

ii) Obtain the power spectral density function of the output process  $\{Y(t)\}$  corresponding to the input process  $\{X(t)\}$  is the system that has an impulse response  $h(t) = e^{-\beta t} U(t)$ . (8)

(OR)

b) A random process X(t) is the input to a linear system whose impulse response is  $h(t) = 2e^{-t}$ ,  $t \ge 0$ . If the autocorrelation function of the process is  $R_{XX}(\tau) = e^{-2|\tau|}$ , determine the following:

The cross correlation function between the input process X(t) and the output process Y(t).