b) The R and D Manager of an automobile company wishes to study the effect of the tyre brand on the tread loss (in millimeter) of tyres. Four different tyres of four different brands (A, B, C and D) are fitted at four different positions of four different cars because there might be variability from one car to another car as well as from one position to another position which is a Latin square design. The data as per this design are presented in the following table. **(16)**

Position (i)	Car (k) 1	2	3	4	Row total
1	A = 8	B = 7	C = 1	D = 7	23
2	B = 11	C = 2	D = 7	A = 8	28
3	C = 6	D = 8	A = 6	B = 6	26
4	D = 4	A = 2	B = 3	C = 8	17
	29	19	17	29	Y = 94

Write the corresponding model. Check whether each component of the model has effect on the tread loss of tyres at a significance level of 5%.

15. a) The data shown in the following table are $\bar{x}$ and R values for 10 samples of size n = 5 taken from a process producing bearings. The measurements are made on the inside diameter of the bearing, with only the last three decimals recorded (i.e., 34.5 should be 0.50345). Set up $\bar{x}$ and R charts on this process. Does the process seem to be in statistical control ? **(16)**

Sample No.	1	2	3	4	5	6	7	8	9	10
Mean($\bar{x}$)	34.5	34.2	31.6	31.5	35.0	34.1	32.6	33.8	34.8	33.6
Range (R)	3	4	4	4	5	6	4	3	7	8

(Conversion factors for n = 5 are $A_2 = 0.577$, $D_3 = 0$ and $D_4 = 2.115$).

(OR)

b) i) The number of weekly customer complaints is monitored at a large hotel using a c-chart. Complaints have been recorded over the past 15 weeks. Develop three-sigma control limits using the following data : **(8)**

Week	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Number of complaints	3	2	3	1	3	3	2	1	3	1	3	4	2	1	1

ii) Thirty-five successive samples of 100 castings each, taken from a production line, contained, respectively, 3, 3, 5, 3, 5, 0, 3, 2, 3, 5, 6, 5, 9, 1, 2, 4, 5, 2, 0, 10, 3, 6, 3, 2, 5, 6, 3, 3, 2, 5, 1, 0, 7, 4 and 3 defectives. If the fraction defective is to be maintained at 0.02, construct a fraction defective chart for these data and state whether or not this standard is being met. **(8)**

08/05/18 FN

Reg. No. : | | | | | | | | | | | | |

Question Paper Code : 41319

B.E./B.Tech. DEGREE EXAMINATION, APRIL/MAY 2018

Third/Fourth/Sixth/Eighth Semester

Environmental Engineering

MA 6468 – PROBABILITY AND STATISTICS

(Common to : Mechanical Engineering (Sandwich)/Agriculture Engineering/ Electrical and Electronics Engineering/Industrial Engineering/Industrial Engineering and Management/Manufacturing Engineering/Bio Technology/ Chemical Engineering/Fashion Technology/Food Technology/Handloom and Textile Technology/Petrochemical Technology/Petroleum Engineering/ Pharmaceutical Technology/Plastic Technology/Polymer Technology/Textile Chemistry/Textile Technology)

(Regulations 2013)

Time : Three Hours Maximum : 100 Marks

Answer ALL questions

PART – A **(10×2=20 Marks)**

1. Suppose that the error in the reaction temperature, in °C, for a controlled laboratory experiment is a continuous random variable X having the probability density function $f(x) = \begin{cases} \frac{x^2}{3}, & -1 < x < 2, \\ 0, & \text{elsewhere.} \end{cases}$ Verify that f(x) is a density function. Find $P(0 < X \leq 1)$.
2. For a certain manufacturing process, it is known that , on the average, 1 in every 100 items is defective. What is the probability that the fifth item inspected is the first defective item found ?
3. Determine the value of k for which the function given by f(x, y) = kxy for x = 1, 2, 3; y = 1, 2, 3 can serve as a joint probability distribution.
4. State the central limit theorem.
5. Write the sampling distribution of the variance.
6. Define type I error and type II error in testing of hypothesis with examples.
7. Write the analysis of variance (ANOVA) table of completely randomized design.
8. What is 2^2 – factorial design ?
9. Why do we need the control charts ? What are the types of control charts ? Give an example for each.
10. What is acceptance sampling ?

PART – B (5×16=80 Marks)

11. a) i) Find the mean and variance of the binomial distribution using moment generating function. (8)

ii) An electrical firm manufactures light bulbs that have a life, before burn-out, that is normally distributed with mean equal to 800 hours and a standard deviation of 40 hours. Find the probability that a bulb burns between 778 and 834 hours. (8)

(OR)

b) i) In a certain industrial facility, accidents occur infrequently. It is known that the probability of an accident on any given day is 0.005 and accidents are independent of each other. What is the probability that in any given period of 400 days there will be an accident on one day ? What is the probability that there are at most three days with an accident ? (8)

ii) Suppose that a system contains a certain type of component whose time, in years, to failure is given by T. The random variable T is modeled nicely by the exponential distribution with mean time to failure $\beta = 5$. If 5 of these components are installed in different systems, what is the probability that at least 2 are still functioning at the end of 8 years ? (8)

12. a) i) Given the joint probability density function

$$f(x,y)=\begin{cases}\frac{3}{5}x(y+x) & \text{for } 0<x<1,\ 0<y<2\\ 0 & \text{elsewhere}\end{cases}$$ of two random variables X and Y,

find $P[(X, Y)\in A]$, where A is the region $\left\{(x,y)\middle|0<x<\frac{1}{2}, 1<y<2\right\}$. (8)

ii) Find the covariance of the random variables whose joint probability density is given by $f(x,y)=\begin{cases}2 & \text{for } x>0,\ y>0,\ x+y<1\\ 0 & \text{elsewhere.}\end{cases}$ (8)

(OR)

b) i) Given the joint probability density function

$$f(x,y)=\begin{cases}4xy & \text{for } 0<x<1,\ 0<y<1\\ 0 & \text{elsewhere}\end{cases}$$ find the marginal densities of X and Y and the conditional density of X given Y = y. (8)

ii) Assume that the random variable S_n is the sum of 48 independent experimental values of the random variable X whose probability density function is given by $f(x)=\begin{cases}\frac{1}{3} & 1\le x\le 4,\\ 0 & \text{otherwise.}\end{cases}$ Find the probability that S_n lies in the range $108 \le S_n \le 126$. (8)

13. a) i) The specifications for a certain kind of ribbon call for a mean breaking strength of 180 pounds. If five pieces of the ribbon (randomly selected from different rolls) have a mean breaking strength of 169.5 pounds with a standard deviation of 5.7 pounds, test the null hypothesis $\mu = 180$ pounds against the alternative hypothesis $\mu < 180$ pounds at the 0.01 level of significance. (8)

ii) The lapping process which is used to grind certain silicon wafers to the proper thickness is acceptable only if σ, the population standard deviation of the thickness of dice cut from wafers, is at most 0.50 mil. Use the 0.05 level of significance to test the null hypothesis $\sigma = 0.50$ against the alternative hypothesis $\sigma > 0.50$, if the thicknesses of 15 dice cut from such wafers have a standard deviation of 0.64 mil. (8)

(OR)

b) For the radio message data below, use goodness of fit to test at the 0.01 level of significance whether the data can be looked as values of a random variable having the Poisson distribution with mean $\lambda = 4.6$. (16)

Number of radio messages	0	1	2	3	4	5	6	7	8	9	10	11	12	13
Observed frequencies	3	15	47	76	68	74	46	39	15	9	5	2	0	1

14. a) There are four different technological alternatives to manufacture a product. The R and D manager of a company feels that the type of technology may have some impact on the hourly output (in units) of the product. Because there might be variability from one plant to another plant, he decides to use the randomized complete block design. The corresponding data are presented in the following table. Write the corresponding model. Check whether each component of the model has effect on the output of the product at a significance level of 5%. (16)

Plant		**Technology**			
		T_1	T_2	T_3	T_4
	P_1	73	68	74	71
	P_2	73	57	75	52
	P_3	45	38	68	40
	P_4	73	41	75	75

(OR)